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BADANIE STYKU WEWNĘTRZNEGO ELEMENTÓW CYLINDRYCZNYCH O PORÓWNYWALNYCH ŚREDNICACH Z MAŁYMI ODCHYŁKAMI OD KSZTAŁTU KOŁOWEGO

INNER CONTACT INVESTIGATION OF CYLINDERS HAVING COMPARABLE DIAMETERS IN CASE OF SMALL OUT-OF-ROUNDNESS

W niniejszej pracy przedstawiono wyniki badań styku wewnętrznego elementów cylindrycznych z małymi odchyłkami od kształtu kołowego. Wszystkie znane prace z tej dziedziny nie podejmują problemu wpływu małych odchyłek od kształtu kołowego. Wyprowadzone zostało podstawowe równanie opisujące ten styk i przedstawiono przybliżone jego rozwiązanie metodą kollokacji. Obliczenia numeryczne przeprowadzone zostały dla typowych odchyłek od kształtu kołowego jak np. odchyłek eliptyczności i różnych rodzajów graniastości, a wyniki przedstawiono w postaci wykresów.

Odchyłki od kształtu kołowego mają istotny wpływ na wartości i rozkład nacisków. Wielkości odchyłek od kształtu kołowego przyjmują wartości przemieszczeń sprężystych, a nawet je przewyższają co zmienia zasadniczo idealizowany schemat styku. Dlatego ilościowa ocena tego wpływu jest ważna ze względów praktycznych.

Otrzymane wyniki obliczeń wskazują na wpływ niedokładności wykonawczych na podstawowe charakterystyki wytrzymałościowe, znacząco się różniących od wyników otrzymanych w/g klasycznej teorii Hertza.

Słowa kluczowe: *kontakt mechaniczny, elementy cylindryczne, tolerancja okrągłości*

In the paper contact problem of the cylindrical elements having small executory deviations from circular shape is investigated. All known solutions of such contact problems do not take into account small deviations from circular shape.

Basic eqation describing the problem is introduced and its asymptotic solution obtained by collocation method is presented. Numerical calculations of quantities characteristing contact for typical deviations from circular shape, like ellipticity, trilobing and tetralobing were presented in graphical way.

Out – of – roundness of element contours has an effect on magitude and distribution of the contact pressurer. Admissible values of out – of – roundnesses are comparable with elastic strains of bodies in contact, and even exceed them, what changes idealised scheme of contact mating. Therefore quantitative estimation of that effect is very important for practical reasons. Obtained numerical results point to greate influence of inaccuracy during production on main quantities characterizing contact, compared with results of classic Hertz problem.

Keywords: *mechanical contact, cylindrical elements, deviations from circular shape, non-Hertzian contact, collocation method*

1. Introduction

In contact strength problem of the cylindrical joints (slide bearings, bolted and articulated joints, guides, chains, brakes, chucks), computational scheme for circular elements [2,4,5] is used. All well-known solutions of such contact problems do not take into account small deviations from circular shape (ellipticity, ovality, lobing) resulting from inaccuracy during production of mentioned joints. Out-of-roundness of element contours has an effect on magnitude and distribution of the contact pressures. Admissible values of out-of-roundnesses are comparable with elastic strains of bodies in contact, and even exceed them, what changes idealised scheme of contact mating. Therefore quantitative estimation of that effect is very important for practical reasons. Problem was stated in the same way in monograph [3], where basis of its solution well-known earlier [1,2], were described in details. In case of ideal contours, problems of the contact strength are the classic ones [2 and others].

In the present article, only authors original results were presented. Quotation of earlier known solutions was limited to indispensable minimum.

2. Problem formulation

Let us consider the following plane contact problem of linear theory of elasticity. In the hole of elastic infinite plane isotropic shield 1, the elastic disk 2 is situated (Fig.1). Contours L_1 (hole) and L_2 (disk) differ somewhat from circles $L^{(1)}$ and $L^{(2)}$ having similar nominal radii R_1 and R_2. Characteristics $\delta_k(\alpha) << R_k$ (Fig.1) of initial out-of roundness forming of owning of inaccuarcy execution may be presented in following way:

$$\delta_k(\alpha) = (-1)^k \left[R^{(k)}(\alpha) - R_k \right] \qquad (1)$$

where: α - polar coordinate of the contour point, $R^{(k)}(\alpha)$ - radius-vector contour of the hole ($k=1$) and the disk ($k=2$).

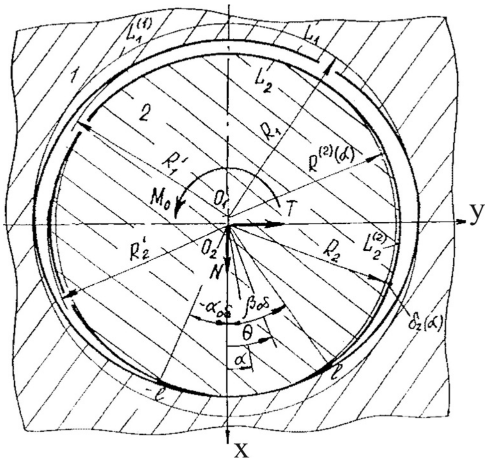

Fig. 1. Computational scheme of the cylindrical joint for elements with small out-of-roundness

Interaction of the elements in contact is caused by static forces N, T and couple of forces having moment M_0. In the contact zone, normal contact stresses $\sigma_r(\alpha, \delta)$ and tangent ones $\tau_{r\alpha}(\alpha, \delta)$ occur. Problem consists in determining distribution of the radial contact pressures $p(\alpha, \delta) = -\sigma_r(\alpha, \delta)$ and the contact zone limited by the angles $\alpha_{o\delta}$ and $\beta_{o\delta}$ (Fig.1), assuming that elements 1 and 2 remain in static equilibrium, and in the zone of point of junction $-\alpha_{o\delta} \le \alpha \le \beta_{o\delta}$ take place contact of outlines L_1 and L_2 without separations.

Boundary conditions on hole and the disk contours take the form:

$$\begin{gathered} \tau_{r\alpha}^{(1)}(\alpha,\delta) = \tau_{r\alpha}^{(2)}(\alpha,\delta) = 0, \\ \sigma_r^{(1)}(\alpha,\delta) = \sigma_r^{(2)}(\alpha,\delta) = 0 \\ (\beta_{o\delta} \le \alpha \le 2\pi - \alpha_{o\delta}); \\ \tau_{\rho\alpha}^{(1)}(\alpha,\delta) = \tau_{\rho\alpha}^{(2)}(\alpha,\delta) = -\phi \cdot \pi(\alpha,\delta), \\ \sigma_\rho^{(1)}(\alpha,\delta) = \sigma_\rho^{(2)}(\alpha,\delta) = -\pi(\alpha,\delta), \\ (-\alpha_{o\delta} \le \alpha \le \beta_{o\delta}) \end{gathered} \qquad (2)$$

where: f - coefficient of sliding friction.

3. Derivation of basic equation of the problem

Let us assume that in contact zone $-\alpha_{o\delta} \le \alpha \le \beta_{o\delta}$ contact of contours L_1 and L_2 without separation appears. Equation for $p(\alpha, \delta)$ has been established using condition of curvatures equality for strained element contours 1 and 2 in the contact zone

$$K_{1*}(\alpha,\delta) = K_{2*}(\alpha,\delta) \qquad (3)$$

Contours parametric equations of L_1 (hole) and L_2 (disk) after straining are as follows:

$$\begin{gathered} x_k(\alpha,\delta) = x_k(\alpha) + u(\alpha) + f_{kx}(\alpha) \\ y_k(\alpha,\delta) = y_k(\alpha) + v(\alpha) + f_{ky}(\alpha) \end{gathered} \qquad (4)$$

where : $x_k(\alpha)$, $y_k(\alpha)$ - components of the initial circles before strain, $k=1$ - circle circumscribed round the hole contour, $k=2$ - circle inscribed in the disk contour; $u(\alpha)$, $v(\alpha)$ - projections of displacement vectors for contour points of elements in contact on axes Ox and Oy; $f_{kx}(\alpha)$, $f_{ky}(\alpha)$ - parameters describing deviation of contour L_1 (hole , $k=1$) and L_2 (disk, $k=2$) from given principal circles.

Introducing (4) into well-known formula for curvature of the flat line and omitting quantities of higher order $(u')^2$, $(v')^2$, f^2_{kx}, f^2_{ky}, we obtain formula for real contour curvatures (with deviations):

$$K_{k*}(\alpha,\delta) = K_{ko}(\alpha) + K_k(\alpha,\delta) \qquad (5)$$

where:

$$K_{ko}(\alpha) = \frac{1}{\left[\left(x_k'\right)^2 + \left(y_k'\right)^2\right]^{3/2}} \Big[x_k' y_k'' - x_k'' y_k' + x_k' v_k'' + y_k'' u_k' - x_k'' v_k' - y_k' u_k'' - 3\frac{x_k' y_k'' - x_k'' y_k'}{\left(x_k'\right)^2 + \left(y_k'\right)^2}\left(x_k' u_k' + y_k' v_k'\right) \Big], \qquad (6)$$

$$K_k(\alpha,\delta) = \frac{1}{\left[\left(x_k'\right)^2 + \left(y_k'\right)^2\right]^{3/2}} \Big[x_k' f_{ky}'' - x_k'' f_{ky}' - y_k'' f_{kx}' - y_k' f_{kx}'' - 3\frac{x_k' y_k'' - x_k'' y_k'}{\left(x_k'\right)^2 + \left(y_k'\right)^2}\left(x_k' f_{kx}' + y_k' f_{ky}'\right) \Big]. \qquad (7)$$

Signs ‘ and ” denote α derivatives.

In case of classic contact of elements with round contours, while determining governing equation, derivatives of displacement vector components $u(\alpha)$, $v(\alpha)$ for contour points of the hole and the disk should be expressed by sought contact stresses $\sigma_r(\alpha)$. Governing equation for $p(\alpha) = -\sigma_r(\alpha)$ is obtained according to (6) and using equality curvature of real outlines (3), which at $K_k(\alpha,\delta)=0$ takes the form $K_{10}(\alpha)=K_{20}(\alpha)$.

It has been assumed that contours L_1 and L_2 differ somewhat from circles. One may prove that in case of elastic strain, projections of the displacement vectors $u(\alpha)$, $v(\alpha)$ of these contours differ from projections of the displacement vectors of adequate points on initial circles in quantity of higher order $f^2_{kx}(\alpha)$, $f^2_{ky}(\alpha)$. Therefore to simplify the problem, we replace the displacement vectors of body contours by the displacement vectors of their initial circles. Expressing derivatives (6) of vector components by contact stresses in [1] and using (3), we obtain equation enabling evaluating mentioned contact stresses. Assuming that $\tau(\alpha,\delta)=f\sigma_r(\alpha,\delta)$, we obtain equation for $p(\alpha,\delta)$ [3]:

$$k_1\int_{-\alpha_{o\delta}}^{\beta_{o\delta}} ctg\frac{\alpha-\theta}{2}\left[p'(\theta,\delta)+fp(\theta,\delta)\right]d\theta = k_2\left[p(\alpha,\delta)-fp'(\alpha,\delta)\right]+$$
$$+k_3\int_{-\alpha_{o\delta}}^{\beta_{o\delta}} p(\alpha,\delta)d\alpha + k_4\left(N\cos\alpha + T\sin\alpha\right)+\frac{R_1-R_2}{R_1R_2}-$$
$$-\sin\alpha\left\{2\left[\frac{f'_{1x}(\alpha)}{R_1^2}-\frac{f'_{2x}(\alpha)}{R_2^2}\right]-\left[\frac{f''_{1y}(\alpha)}{R_1^2}-\frac{f''_{2y}(\alpha)}{R_2^2}\right]\right\}+$$
$$+\cos\alpha\left\{2\left[\frac{f'_{1y}(\alpha)}{R_1^2}-\frac{f'_{2y}(\alpha)}{R_2^2}\right]+\left[\frac{f''_{1x}(\alpha)}{R_1^2}-\frac{f''_{2x}(\alpha)}{R_2^2}\right]\right\} \qquad (8)$$

where: $k_1 = \frac{1}{8\pi}\left(\frac{1+\kappa_1}{G_1R_1}+\frac{1+\kappa_2}{G_2R_2}\right)$; $k_2 = \frac{1}{4}\left(\frac{1-\kappa_1}{G_1R_1}-\frac{1-\kappa_2}{G_2R_2}\right)$;

$k_3 = \frac{1+\kappa_1}{8\pi G_1R_1}$; $k_4 = \frac{1}{2\pi}\left(\frac{\kappa_1}{G_1R_1}+\frac{1}{G_2R_2}\right)$; $p'(\theta,\delta) = dp(\theta,\delta)/d\theta$;

$$\begin{Bmatrix}N\\T\end{Bmatrix} = -\int_{-\alpha_{o\delta}}^{\beta_{o\delta}}\left[\sigma_r^{(2)}(\alpha,\delta)\begin{Bmatrix}\cos\alpha\\\sin\alpha\end{Bmatrix}\mp\tau_{r\alpha}^{(2)}(\alpha,\delta)\begin{Bmatrix}\sin\alpha\\\cos\alpha\end{Bmatrix}\right]d\alpha$$

$$f_{kx}(\alpha) = x_k^*(\alpha,\delta)-x_k(\alpha),$$
$$f_{ky}(\alpha) = y_k^*(\alpha,\delta)-y_k(\alpha); \qquad (9)$$

where : $\kappa=3-4\nu$ - for plane state of strain; $\kappa=(3-\nu)/(1+\nu)$ - for plane state of stress; ν - Poisson ratio, G - shear modulus, x^*_k, y^*_k - parametric equations of elements out – for displacements f round contours before strain.

4. Contact of cylinders with ellipticity

Deviations from circular shape may be described in different way. The simplest description presents equation of the out-of-round contour in the canonical form, for example as an ellipse equation. We assume that friction does not occur in the contact zone ($f=0$) and elements are made of the same materiel ($G_1=G_2$, $\chi_1=\chi_2$) . Contact mating of elements in the joint is determined only by force N, hence $\alpha_{o\delta}=\beta_{o\delta}$. Semi-axes of elliptic elements (Fig.1) are equal : $a_1=R_1$, $b_1=R'_1$, $a_2=R'_2$, $b_2=R_2$, and $a_1>b_1$, $a_2>b_2$. Elements radii are similar, i.e. , but radial clearance $\varepsilon=R_1-R_2>0$.

Parametric equations of the elliptic elements contours (placed like in Fig.1) are described by well-known formulas :

$$x_k^*(\alpha,\delta) = X_k\cos\alpha\ ;\ y_k^*(\alpha,\delta) = Y_k\sin\alpha \qquad (10)$$

for $k=1$: $X_1=b_1$, $Y_1=a_1$; for $k=2$: $X_2=b_2$, $Y_2=a_2$.

Equation of the initial circles :

$$x_k(\alpha) = A_k\cos\alpha\ ;\ y_k(\alpha) = A_k\sin\alpha \qquad (11)$$

for $k=1$, $A_1=R_1$; for $k=2$, $A_2=R_2$.

Taking (9), (10), (11) and admitted assumptions into account, equation for considered case takes the form :

$$\frac{1}{\pi}\int_{-\alpha_{o\delta}}^{\alpha_{o\delta}} ctg\frac{\alpha-\theta}{2}p\cdot(\theta,\delta)d\theta = \frac{2}{\pi}\cos\alpha\int_{-\alpha_{o\delta}}^{\alpha_{o\delta}} p(\alpha,\delta)\cos\alpha\, d\alpha +$$
$$+\frac{1}{2\pi}\int_{-\alpha_{o\delta}}^{\alpha_{o\delta}} p(\alpha,\delta)d\alpha + \frac{4\varepsilon G}{R(1+\kappa)}\left[1-\frac{\varepsilon_1}{\varepsilon}D_1(\alpha)-\frac{\varepsilon_2}{\varepsilon}D_2(\alpha)\right] \qquad (12)$$

where : $D_1(\alpha)=\cos^2\alpha$, $D_2(\alpha)=\sin^2\alpha$, $\varepsilon_1=\delta_1-2\delta_2$, $\varepsilon_2=2\delta_1-\delta_2$ - characterise contour shape deviations and their mutual orientation; $\delta_1=a_1-b_1$, $\delta_2=a_2-b_2$ - maximal deviations of elliptic elements from circles $\delta_1\le\varepsilon$, $\delta_2\le\delta_1$.

In the second joint scheme two elements contact each other by the vertices in the direction of longer semi-axes. Then in (10) for $k=1$: $X_1=a_1$, $Y_1=b_1$; for $k=2$; $X_2=a_2$, $Y_2=b_2$ and in (12) $\varepsilon_1=\delta_2-2\delta_1$, $\varepsilon_2=2\delta_2-\delta_1$ and $\delta_1\le\delta_2$, $\delta_2\le\varepsilon$.

In the third joint scheme the hole is situated like in the Fig.1 and the disk touches its contour vertically by the ellipse vertex. Then for $k=1$: $X_1=b_1$, $Y_1=a_1$; for $k=2$: $X_2=a_2$, $Y_2=b_2$; $\varepsilon_1=(\delta_1+\delta_2)$, $\varepsilon_2=2(\delta_1+\delta_2)$, and $|\varepsilon_1|\le\varepsilon$.

Equation (12) is solved using approximate collocation method and distribution of the contact pressures p(, δ) is chosen in the following form [2]:

$$p(\alpha,\delta)\cong\left(C_o+C_2tg^2\frac{\alpha}{2}\right)\sqrt{tg^2\frac{\alpha_{o\delta}}{2}-tg^2\frac{\alpha}{2}} \qquad (13)$$

where: C_O, C_2 - collocations coefficients.

Substituting in (12) $tg\alpha/2=\xi$, $tg\theta/2=z$, we obtain

$$\frac{1}{\pi}\int_{-d_o}^{d_o}\frac{p'(2arctgz)}{z-\xi}dz = -\frac{2}{\pi}\frac{1-\xi^2}{1+\xi^2}\int_{-d_o}^{d_o}\frac{(1-\xi^2)p(2arctg\xi)}{(1+\xi^2)^2}d\xi -$$
$$\frac{1}{2\pi}\int_{-d_0}^{d_0}\frac{p(2arctg\xi)}{1+\xi^2}d\xi+\frac{1}{\pi}\int_{-d_o}^{d_o}\frac{\xi p'(arctg\xi)}{1+\xi^2}d\xi$$
$$-\frac{2G\varepsilon}{R(1+\kappa)}\left[1-\frac{\varepsilon_1}{\varepsilon}D_1(\xi)-\frac{\varepsilon_2}{\varepsilon}D_2(\xi)\right] \qquad (14)$$

where: $d_o=tg\alpha_{o\delta}/2$.

Integrating (14), we obtain equations which enable determination of un-known coefficients C_o, C_2:

$$C_o A_{o1}(\xi_1) + C_2 A_{21}(\xi_1) = -\frac{\varepsilon G}{R(1+\kappa)}\left[1 - \frac{\varepsilon_1}{\varepsilon} D_1(\xi_1) - \frac{\varepsilon_2}{\varepsilon} D_2(\xi_1)\right]$$

$$C_o A_{o2}(\xi_2) + C_2 A_{22}(\xi_2) = -\frac{\varepsilon G}{R(1+\kappa)}\left[1 - \frac{\varepsilon_1}{\varepsilon} D_1(\xi_2) - \frac{\varepsilon_2}{\varepsilon} D_2(\xi_2)\right] \quad (15)$$

where: $\zeta_1=0$, $\zeta_2=0,65d_o$ - collocation nodes,

$$C_o = \frac{A_{21}(\xi_1) K_2(\xi_2,\delta) - A_{22}(\xi_2) K_1(\xi_1,\delta)}{A_{o1}(\xi_1) A_{22}(\xi_2) - A_{o2}(\xi_2) A_{21}(\xi_1)} \frac{\varepsilon G}{R(1+\kappa)}$$

$$C_2 = \frac{A_{o2}(\xi_2) K_1(\xi_1,\delta) - A_{o1}(\xi_1) K_2(\xi_2,\delta)}{A_{o1}(\xi_1) A_{22}(\xi_2) - A_{o2}(\xi_2) A_{21}(\xi_1)} \frac{\varepsilon G}{R(1+\kappa)} \quad (16)$$

where:

$$K_1(\xi_1,\delta) = 1 - \frac{\varepsilon_1}{\varepsilon} D_1(\xi_1) - \frac{\varepsilon_2}{\varepsilon} D_2(\xi_1)$$

$$K_2(\xi_2,\delta) = 1 - \frac{\varepsilon_1}{\varepsilon} D_1(\xi_2) - \frac{\varepsilon_2}{\varepsilon} D_2(\xi_2)$$

$$A_{oj}(\xi_j) = -0,25\left(1+\xi_j^2\right) + 0,25\left(\tilde{b}-1\right) + \left(\tilde{b}-1\right)\left(1-\xi_j^2\right)\left[\tilde{b}\left(1+\xi_j^2\right)\right]^{-1}$$

$$A_{2j}(\xi_j) = 0,125\left(d_o - 6\xi_j^2\right)\left(1+\xi_j^2\right) + 0,125\left(\tilde{b}-1\right)^2 +$$

$$+\left[4d_o^2 - \left(d_o^2+6\right)\tilde{b} + 6\right]\left(1-\xi_j^2\right)\left[2\tilde{b}\left(1+\xi_j^2\right)\right]^{-1} \quad j=1,2 \quad (17)$$

Quantities $D(\xi_j)$, $D_2(\xi_j)$, taking change of variables according to (12) into account, are determined as follows:

$$D_1(\xi_j) = \frac{\left(1-\xi_j^2\right)^2}{\left(1+\xi_j^2\right)^{-2}}, \quad D_2(\xi_j) = \frac{4\xi^2}{\left(1+\xi_j^2\right)^{-2}} \quad (18)$$

In the case when $G_1 \neq G_2$ and $\kappa_1 \neq \kappa_2$, term $G/(1+\kappa)$ has not been into account in (16), but they will be used in $A_{oj}(\xi_j)$, $A_{2j}(\xi_j)$. Respectively:

$$A_{oj}(\xi_j) = \pi R\left\{-k_1\left(1+\xi_j^2\right) + \frac{k_2}{\pi}\sqrt{d_o^2-\xi_j^2} + 2k_3\left(\tilde{b}-1\right) + \right.$$

$$\left. + 2k_4\left(\tilde{b}-1\right)\cdot\left(1-\xi_j^2\right)\left[\tilde{b}\left(1+\xi_j^2\right)\right]^{-1}\right\}$$

$$A_{2j}(\xi_j) = \pi R\left\{\frac{k_1}{2}\left(d_o^2 - 6\xi_j^2\right)\left(1+\xi_j^2\right) + \frac{k_2}{\pi}\xi_j^2\sqrt{d_o^2-\xi_j^2} + k_3\left(\tilde{b}-1\right)^2 + \right.$$

$$\left. k_4\left[4d_o^2 - \left(d_o^2+6\right)\tilde{b} + 6\right]\left(1-\xi_j^2\right)\left[2\tilde{b}\left(1+\xi_j^2\right)\right]^{-1}\right\}$$

The contact angle will be determined using equilibrium condition of forces applied on the disk:

$$N = R\int_{-\alpha_{o\delta}}^{\alpha_{o\delta}} p(\alpha,\delta)\cos\alpha \, d\alpha \quad (19)$$

Substituting $p(\alpha,\delta)$ according to (13) and integrating, we obtain equation for $\alpha_{0\delta}$ in the form:

$$N = 2\pi R\left(C_o\bar{A}_o + C_2\bar{A}_2\right) \quad (20)$$

where:

$$\bar{A}_o = 1 - \tilde{b}^{-1}, \quad \bar{A}_2 = 0,5\tilde{b}^{-1}\left[d_o^2\left(4-\tilde{b}\right) + 6\left(1-\tilde{b}\right)\right]$$

Equations system (15) is solved together with equation (20) in order to search roots for assumed N.

5. Simplified way of characteristics calculation for contact of round and prismatic elements

Elements of cylindrical joints are manufactured with particular tolerance. Therefore, the radial clearance ε is different in individual joint. Method, presented above, enables to determine magnitude of the maximal pressures $p(0)$ or $p(0,\delta)$ and their distribution (angle α_o or $\alpha_{o\delta}$) for each ε. Analysing numerical solutions, one can draw conclusion that under assumption R = const and N = const, values of $p(0)$, $p(0, \delta)$ and α_o, $\alpha_{o\delta}$ when clearances ε change may be evaluated easily.

In the joints of round elements, $p(0)$ and α_o may be evaluated from

$$p(0)_{\varepsilon^-} \cong p(0)_{\varepsilon^+}\tilde{\varepsilon}_p \quad (21)$$

$$\alpha_{o\varepsilon^-} \cong \alpha_{o\varepsilon^+}\tilde{\varepsilon}_\alpha \quad (22)$$

here: $p(0)_{\varepsilon^+}$ - known value of the maximal contact pressure $p(0)$ for certain value of clearance $\varepsilon=\varepsilon^+$ determined according to given method; $p(0)_{\varepsilon^-}$ -searched the maximal contact pressure for other value of clearance $\varepsilon=\varepsilon^- \neq \varepsilon^+$; denotations for the angles $\alpha_{o\varepsilon^+}$ and $\alpha_{o\varepsilon^-}$ are similar; $\tilde{\varepsilon}_p = \sqrt{\varepsilon^-/\varepsilon^+}$, $\tilde{\varepsilon}_\alpha = \tilde{\varepsilon}_p^{-1} = \sqrt{\varepsilon^+/\varepsilon^-}$

For the joints of prismatic elements, formulas (21), (22) take the form:

$$p(0)_{\delta\varepsilon^-} \cong p(0)_{\varepsilon^-}\tilde{\varepsilon}_{p\delta} \cong p(0)_{\varepsilon^+}\tilde{\varepsilon}_p\tilde{\varepsilon}_{p\delta} \quad (23)$$

$$\alpha_{o\delta\varepsilon^-} \cong \alpha_{o\varepsilon^-}\tilde{\varepsilon}_{\alpha\delta} \cong \alpha_{o\varepsilon^+}\tilde{\varepsilon}_\alpha\tilde{\varepsilon}_{\alpha\delta} \quad (24)$$

where: $\tilde{\varepsilon}_{\alpha\delta} = \tilde{\varepsilon}_{p\delta}^{-1}$.

Comparison of the contact pressures magnitude and the contact areas, calculated according to presented method and formulas (21) - (24), shows their good correlation for $0.05 \leq \varepsilon \leq 0.4$ *mm*. In the case when $\varepsilon < 0.5$ *mm* the difference between them increases visibly.

6. Joints with ellipticity. Numerical solution and analysis of results

For the case of plane state of strain, calculations of the contact characteristics were carried out for the following data: ε =*0.05*; *0.1*; *0.2*; *0.4* mm; δ_1 and δ_2 = *0.05; 0.1; 0.2; 0.3; 0.4* mm; ν=*0.3*; *N=0.1; 1.0, 5.0* MN; *R=0.1* m; $G=8.1\ 10^4$ MPa.

Calculation results for chosen cases are presented as variation Plotss (Fig. 2,3,4) $\tilde{p} = p(0,\delta)/p(0)$ of the maximal pressures $p(0,\delta)$ according to the ellipticity δ_1, δ_2, radial clearance ε and load N. Maximal pressures for contact of the circular bodies are denoted by p(0). The mentioned-below table presents denotations for Fig.2 and the next ones.

Figs.2 and 3 show $\tilde{p} \sim \delta_2$ plots for assumed ε ($\varepsilon = 4{\cdot}10^{-4}$ m - thickened solid line, $\varepsilon = 2{\cdot}10^{-4}$ m - thickened dashed line). Fine lines show boundary curves of $\tilde{p}$ variation areas according to the value of clearance ε.

Plotss for the first case (Fig.2) point out that an increase in the value of elements ellipticity results in $\tilde{p}$ for every value of clearance ε. It is worth noticing that relative pressures $\tilde{p}$ decrease for every δ_1=*const* as δ_2 increases.

Table 1

Area denotation $\tilde{p}$	Value of parameter	Number of curve	Value of $_1$ parameter
Σ_1	= 0.4 mm	1	$_1$= 0.4 mm
Σ_2	= 0.2 mm	2	$_1$= 0.3 mm
Σ_3	= 0.1 mm	4	$_1$= 0.2 mm
Σ_4	= 0.05 mm	5	$_1$= 0.1 mm
Σ_5	= 0	6	$_1$= 0.05 mm
		7	$_1$= 0

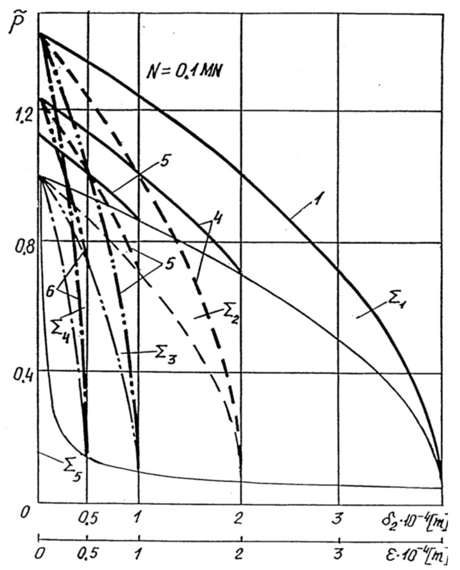

Fig. 2. Plots of relative variations of the maximal pressures for the joint scheme: $\delta_1 \leq \varepsilon$, $\delta_2 \leq \delta_1$($\delta_1 = a_1 - b_1$, $\delta_2 = a_2 - b_2$, $\varepsilon = R_1 - R_2 = a_1 - b_2$)

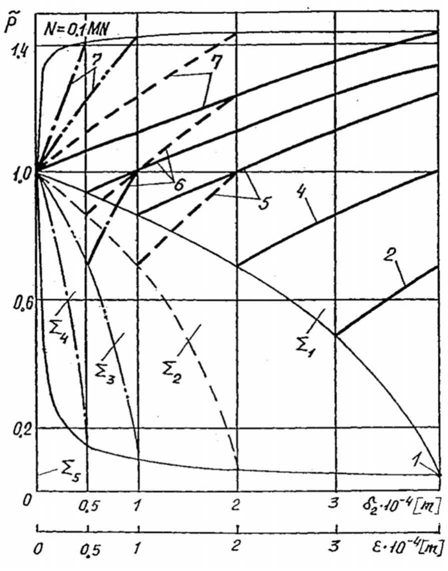

Fig. 3. Plots of relative variations of the maximal pressures for the joint scheme: $\delta_2 \leq \varepsilon$, $\delta_1 < \delta_2$

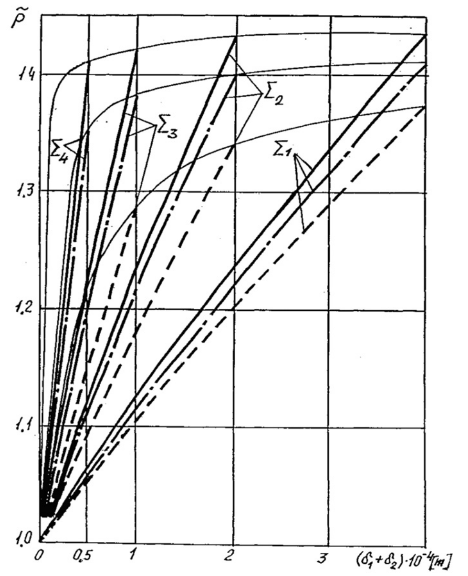

Fig. 4. Plots of relative variations of the maximal pressures for the joint scheme: $\delta_1 + \delta_2 \leq \varepsilon$

In the second case of contact (Fig.2), an increase in the value of δ_1 results in $\tilde{p}$ reduction for δ_2=*const*. For δ_1>*const* an increase in the value of δ_2 results in growth of $\tilde{p}$. Plots shows that-according to δ_1, δ, ε - the contact favourable conditions remain in comparison with the contact of the circular elements, i.e. $\tilde{p}$<1,0 and for other $\tilde{p}$>1,0.

In the third case (Fig.4) the ellipticity of the element contours causes an increase in the pressures $\tilde{p}$. Values of $\tilde{p}$ depend on the sum $\delta_1 + \delta_2$. The influence of load N on the variation of $\tilde{p}$ also was investigated. Solid lines concern case when $N = 0.1$ MN, dot-and-dash lines -when $N = 1.0$ MN and dashed lines -when $N = 5$ MN.

Theoretical investigations concerning contact problem for bodies with small ellipticity testify to considerable influence of out-of-roundness on the contact pressures. According to mutual orientation of elements in the joint, one can observe an increase in *p(0,δ)* (Fig.4), a decrease (Fig.2) or both cases (Fig.3). Therefore it is obvious that deviations of the cylindrical elements from circular shape should be taken into account as a factor which considerably influences magnitude of the contact pressures. Also Figs. 5, 6, 7 show variation Plotss $\bar{\alpha} = \alpha_{o\delta} / \alpha_o$ of the contact semi-angle $\alpha_{o\delta}$ according to δ_1, δ_2, ε. The angle α_o is the semi-angle of cylinders contact without ellipticity. Plotss point out considerable influence of ellipticity on the distribution of the contact pressures.

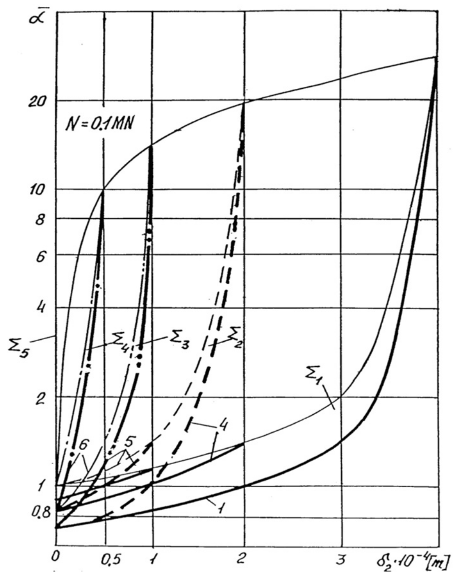

Fig. 5. Plots of relative variations of the contact area for the joint scheme: $\delta_1 \le \varepsilon$, $\delta_2 < \delta_1$

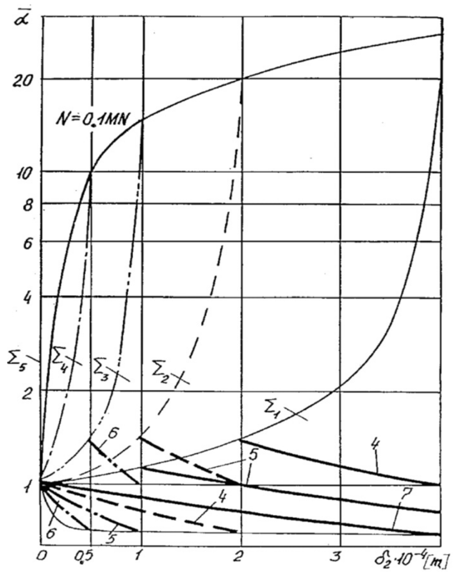

Fig. 6. Plots of relative variations of the contact area for the joint scheme: $\delta_2 \le \varepsilon$, $\delta_1 < \delta_2$

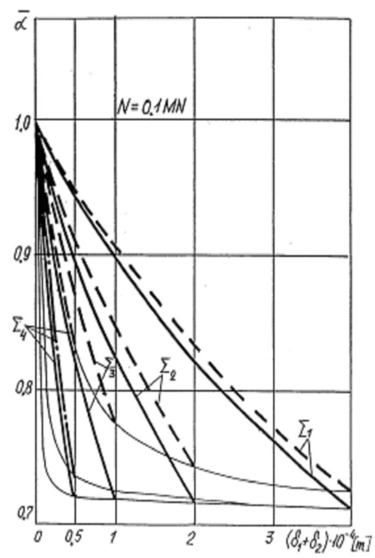

Fig. 7. Plots of relative variations of the contact area for the joint scheme: $\delta_1 + \delta_2 \le \varepsilon$

Obtained results indicate that different kinds of out-of-roundnesses have considerable effect on distribution of the contact pressures. An increase or decrease of the contact zone is possible, depending on mutual orientation of elements in the joint, radial clearance, parameters of contours out-of-roundness and load.

Taking more collocation nodes in formula (13) than two is not necessary. That shows following results (Fig. 1; $\alpha_0 = \frac{5}{18}\pi$) $\frac{p(\alpha)}{\varepsilon E / R}$ for exactly solution (1), solution for two nodes (2) and solution for three nodes (3):

α	(1)	(2)	(3)
0	0,4341	0,4339	0,4339
π/18	0,4249	0,4247	0,4246
π/9	0,3954	0,3953	0,3951
π/6	0,3423	0,3419	0,3420
2π/9	0,2536	0,2525	0,2535
5π/16	0	0	0

Difference between solutions (2) and (3) amounts less than 1% of the exactly results.

9. Conclusions

Calculation results (Fig. 2, 3, 4) presents effect out-of-roundness of element contours, cylindrical joints on magnitude and distribution of the contact pressures.

Plots of relative variations of the contact are shows Fig. 5÷7.

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