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RELIABILITY AND PROFIT ANALYSIS OF A SINGLE-UNIT SYSTEM WITH PREVENTIVE MAINTENANCE SUBJECT TO MAXIMUM OPERATION TIME

ANALIZA NIEZAWODNOŚCI I ZYSKU DLA SYSTEMU JEDNOELEMENTOWEGO Z KONSERWACJĄ ZAPOBIEGAWCZĄ PODDANEGO MAKSYMALNEMU CZASOWI PRACY

This paper deals with the profit analysis of a reliability model for a single-unit system in which unit fails completely either directly from normal mode or via partial failure. The partially failed operating unit is shutdown after a maximum operation time for preventive maintenance. There is a single server who attends the system immediately whenever needed to conduct preventive maintenance at partial failure stage and repair at completely failure stage of the unit. The unit works as new after preventive maintenance and repair. The switch devices are considered as perfect. All random variables are assumed as independent and uncorrelated. The distribution of failure times, maximum operation time, preventive maintenance time and repair time are taken as general. Various reliability characteristics of interest are evaluated by using semi-Markov process and regenerative point technique. The tabular represantation of mean time to system failure (MTSF), availability and profit with respect to maximum rate of operation time has also been shown for a particular case.

Keywords: *single-unit system, reliability, preventive maintenance, maximum operation time, profit analysis.*

W niniejszej pracy przedstawiono analizę zysku modelu niezawodności dla systemu jednoelementowego, w którym element ulega całkowitemu uszkodzeniu bezpośrednio z trybu normalnego lub pośrednio na skutek częściowego uszkodzenia. Częściowo uszkodzona działająca jednostka jest wyłączana po upłynięciu maksymalnego czasu pracy w celu przeprowadzenia konserwacji zapobiegawczej. Pojedynczy serwer wspomaga bezzwłocznie system w momencie wystąpienia potrzeby przeprowadzenia konserwacji zapobiegawczej na etapie częściowego uszkodzenia oraz naprawy na etapie uszkodzenia całkowitego. Element działa jak nowy, po konserwacji zapobiegawczej i naprawie. Stan przełączników sieciowych uznaje się za doskonały. Wszystkie zmienne losowe traktowano jako niezależne i nieskorelowane. Rozkład czasów uszkodzeń, maksymalnego czasu pracy, czasu konserwacji zapobiegawczej i czasu naprawy przyjęto jako ogólne. Wybrane parametry niezawodnościowe oceniano za pomocą procesu semi-markowskiego i techniki odnowy RPT. Dla poszczególnych przykładów przedstawiono także tabelaryczne zestawienie średniego czasu do uszkodzenia systemu (MTSF), gotowości i zysku w odniesieniu do maksymalnego czasu pracy.

Słowa kluczowe: *system jednoelementowy, niezawodność, konserwacja zapobiegawcza, maksymalny czas pracy, analiza zysku.*

1. Introduction

Several researchers including Barlow and Larry [1], Nakagawa and Osaki [13], Murari and Goyal [12], Mokaddis et al. [11], Kumar et al. [6] and Renbin and Zaiming [14] have probed systems of one or more units making the assumption that the operating unit enters directly into the failed stage with constant failure rate and whenever the unit is under operation, it is continued until it fails.

But, in practice, there are many situations where a unit may fail completely either directly from normal mode or via various degraded stages. The devices subject to wear in reliability and the immune system of HIV infected individual in Bio-statistics can be considered as examples of such systems. However, continuous operation of a unit for a long time causes defects in the unit and increases the maintenance cost. Also, the continued operation and ageing of the systems gradually reduce their performance, reliability and safety. It can be seen from literature that preventive maintenance can slow the deterioration process of a repairable system and restore the system to a younger age or state. Therefore, preventive maintenance of the systems is necessary after a pre-specific period of time not only to maintain the operational power but may also reduce the failure and the degradation rate.

Keeping above facts in view, present paper deals with the cost-benefit analysis of a reliability model for a single-unit system in which unit fails completely either directly from normal mode or via partial failure. The partially failed operating unit is shutdown after a maximum operation time for preventive maintenance. There is a single server who attends the system immediately whenever needed to conduct preventive maintenance at partial failure stage and repair at completely failure stage of the unit. The unit works as new after preventive maintenance and repair. The switch devices are considered as perfect. All random variables are assumed as independent and uncorrelated. The distribution of failure times, maximum operation time, preventive maintenance time and repair time are taken as general. Various reliability characteristics for interest are evaluated by using semi-Markov process and regenerative point technique. The tabular represantation of MTSF, availability and profit with respect to maximum rate of operation time has also been shown for a particular case.

2. Notation

E	Set of regenerative states
O	The unit is operative and in normal mode
PFO	The unit is partially failed and operative
PFPm	The unit is partially failed and under preventive
FUr	The unit is failed and under repair
$f(t), F(t)$	Probability desity function (p.d.f.),Cumulative distribution function (c.d.f.) of the failure time from normal mode to complete failure
$f_1(t), F_1(t)$	p.d.f.,c.d.f. of the failure time from normal mode to partial failure
$f_2(t), F_2(t)$	p.d.f.,c.d.f. of the failure time from partial failure to complete failure
$g(t), G(t)$	p.d.f.,c.d.f. of the repair time of a failed unit
$z(t), Z(t)$	p.d.f.,c.d.f. of maximum operation time after partial failure
$h(t), H(t)$	p.d.f.,c.d.f.of the preventive maintenance time of the unit
*	Laplace transforms
©	Convolution

$E_0(t) = \overline{F(t)}\overline{F_1(t)}$ $\quad E_1(t) = \overline{Z(t)}\overline{F_2(t)}$

$E_2(t) = f_2(t)\overline{Z(t)}$ $\quad E_3(t) = z(t)\overline{F_2(t)}$

The system may be in one of the following states:

Up states $S_0(O)$, $S_1(PFO)$, $S_2(PFPm)$ Down states $S_3(FUr)$. Possible transitions between states along with cumulative distribution functions time are shown in Table 1.

Table 1.

From	S_0	S_1	S_2	S_3
S_0	-	$F_1(t)$	$F(t)$	-
S_1	-	-	$F_2(t)$	$Z(t)$
S_2	$G(t)$	-	-	-
S_3	$H(t)$	-	-	-

3. Reliability Analysis

Let $R_i(t)$ as the probability that the system survives during $(0, t) \mid E_0(t) = S_i$. To determine it we regard the failed states as absorbing state. The equations determining the reliability of the system. Hence we have:

$$R_0(t) = E_0(t) + f_1(t) © R_1(t)$$

$$R_1(t) = E_1(t) \tag{3.1}$$

By using Laplace transform technique, we can solve for $R_0^*(s)$ and is given by:

$$R_0^*(s) = E_0^*(s) + f_1^*(s)E_1^*(s) \tag{3.2}$$

The steady-state reliability of the system given by

$$R_0 = \lim_{s\to 0} sR_0^*(s) = \lim_{t\to\infty} R_0(t) \tag{3.3}$$

4. Availability Analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant *t* given that the system entered regenerative state *i* at *t=0*. The recursive relations for $A_i(t)$ are given by:

$$A_0(t) = E_0(t) + f_1(t) © A_1(t)$$

$$A_1(t) = E_1(t) + E_2(t) © A_2(t) + E_3(t) © A_3(t)$$

$$A_2(t) = g(t) © A_0(t)$$

$$A_3(t) = h(t) © A_0(t) \tag{4.1}$$

By taking Laplace transforms of the above equations and solving for $A_0^*(s)$, we get:

$$A_0^*(s) = \frac{N_1(s)}{D(s)} \tag{4.2}$$

where:

$$N_1(s) = E_0^*(s) + f_1^*(s)E_1^*(s)$$

$$D(s) = 1 - f_1^*(s)[h^*(s)E_3^*(s) + g^*(s)E_2^*(s)]$$

The steady-state availability of the system given by:

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \lim_{t\to\infty} A_0(t) \tag{4.3}$$

5. Busy Period of the Server due to Repair

Let $B_i^R(t)$ is defined as the probability that the system is busy due to repair at epoch t starting from state $S_i \in E$ .we have the following recursive relation:

$$B_0^R(t) = f_1(t) © B_1^R(t)$$

$$B_1^R(t) = E_2(t)©B_2^R(t) + E_3(t)©B_3^R(t)$$

$$B_2^R(t) = \overline{G(t)} + g(t)©B_0^R(t)$$

$$B_3^R(t) = h(t)©B_0^R(t) \tag{5.1}$$

By taking Laplace transforms of the above equations and solving for $B_0^{R*}(s)$, we get:

$$B_0^{R*}(s) = \frac{N_2(s)}{D(s)} \tag{5.2}$$

where: $N_2(s) = \overline{G^*(s)} f_1^*(s) E_2^*(s)$

$D(s) = 1 - f_1^*(s)[h^*(s)E_3^*(s) + g^*(s)E_2^*(s)]$

The steady-state of the busy period due to server is given by:

$$B_0^R = \lim_{s\to 0} sB_0^{R*}(s) = \lim_{t\to\infty} B_0^R(t) \tag{5.3}$$

6. Busy Period of the Server due to Preventive Maintenance

Let $B_i^P(t)$ is defined as the probability that the system is busy due to Preventive Maintenance at epoch t starting from state $S_i \in E$ .we have the following recursive relation:

$$B_0^P(t) = f_1(t)©B_1^P(t)$$

$$B_1^P(t) = E_2(t)©B_2^P(t) + E_3^P(t)©B_3^P(t)$$

$$B_2^P(t) = g(t)©B_0^P(t)$$

$$B_3^P(t) = \overline{H(t)} + h(t)©B_0^P(t) \tag{6.1}$$

By taking Laplace transforms of the above equations and solving for $B_0^{P*}(s)$, we get

$$B_0^{P*}(s) = \frac{N_3(s)}{D(s)} \tag{6.2}$$

where:

$N_3(s) = \overline{H^*(s)}\, f_1^*(s)\, E_3^*(s)$

$D(s) = 1 - f_1^*(s)[h^*(s)E_3^*(s) + g^*(s)E_2^*(s)]$

The steady-state of the busy period due to preventive maintenance server is given by:

$$B_0^P = \lim_{s\to 0} sB_0^{P*}(s) = \lim_{t\to\infty} B_0^P(t) \tag{6.3}$$

7. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state *i* at *t=0*. We have the following recursive relations for $N_i(t)$:

$$N_0(t) = f_1(t)©N_1(t)$$

$$N_1(t) = E_2(t)©[1 + N_2(t)] + E_3(t)©[1 + N_3(t)]$$

$$N_2(t) = g(t)©N_0(t)$$

$$N_3(t) = h(t)©N_0(t) \tag{7.1}$$

By taking Laplace transforms of the above equations and solving for $N_0^*(s)$, we get:

$$N_0^*(s) = \frac{N_4(s)}{D(s)} \tag{7.2}$$

where:

$N_4(s) = f_1^*(s)[E_2^*(s) + E_3^*(s)]$

$D(s) = 1 - f_1^*(s)[h^*(s)E_3^*(s) + g^*(s)E_2^*(s)]$

The steady-state of the busy period due o server is given by:

$$N_0 = \lim_{s\to 0} sN_0^*(s) = \lim_{t\to\infty} N_0(t) \tag{7.3}$$

8. Profit Analysis

Any manufacturing industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major factors contributing to the total cost are availability, busy period of server and expected number of visits by the server. The cost of these individual items varies with reliability or mean time to system failure. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.

The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function in time t is given by:

P(t) = Expected revenue in (0, t] – Expected total cost in (0, t]

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as

$$\lim_{t\to\infty} \frac{P(t)}{t}$$

i.e. profit per unit time = total revenue per unit time – total cost per unit time. Considering the various costs, the profit equation is given as:

$$P = K_1 A_0 - K_2 B_0^R - K_3 B_0^P - K_4 N_0$$

where: K_1 = Revenue per unit up-time of the system,
K_2 = Cost per unit time for which server is busy in repair,
K_3 = Cost per unit time for which server is busy in preventive maintenance
K_4 = Cost per unit visit by the server.

9. Numerical Results

In this section,some of the results obtained for the above system are illustrated with a numerical example,we assume that

$$f(t) = \lambda e^{-\lambda t} \qquad f_1(t) = \lambda_1 e^{-\lambda_1 t} \qquad f_2(t) = \lambda_2 e^{-\lambda_2 t}$$

$$g(t) = \theta e^{-\theta t} \qquad h(t) = \beta e^{-\beta t} \qquad z(t) = \alpha e^{-\alpha t}$$

From equation (3.2), the time-dependent reliability is given by:

$$R_0^*(t) = \sum_{i=1}^{3} \frac{[s_i(s_i + \alpha + \lambda_2) + \lambda_1(2s_i + \alpha + \lambda + \lambda_1 + \lambda_2)]e^{s_i t}}{\prod_{j=1, i\neq j}^{3}(s_i - s_j)}$$

where $s_i (i = 1 \; to \; 3)$ are the roots of the given equation.

$$s^3 + s^2(\alpha + \lambda + 2\lambda_1 + \lambda_2) + s(\lambda\alpha + \lambda\lambda_2 + 2\lambda_1\alpha + 2\lambda_1\lambda_2 + \lambda\lambda_1 + \lambda_1^2) + \lambda\lambda_1\alpha + \lambda\lambda_1\lambda_2 + \lambda_1^2\alpha + \lambda_1^2\lambda_2 = 0$$

Hence the mean time to failure of the system is calculated using the relation MTSF= $R_0^*(0) = \dfrac{(\alpha + \lambda_1 + \lambda_2 + \lambda_1)}{\lambda\alpha + \lambda\lambda_2 + \lambda_1\alpha + \lambda_1\lambda_2}$

Now from equation (4.2) the time-dependent availability of the system is given by:

$$A_0^*(t) = \sum_{i=1}^{5} \frac{[(s_i^2 + s_i(\alpha + \lambda_2 + 2\lambda_1) + \alpha\lambda_1 + \lambda_1^2 + \lambda_1\lambda_2 + \lambda\lambda_1)(s_i^2 + s_i\beta + \theta s_i + \theta\beta)]e^{s_i t}}{\prod_{j=1, j\neq i}^{5}(s_i - s_j)}$$

where $s_i (i = 1 \; to \; 5)$ are the roots of the equation

$$\begin{aligned}
&s^5 + s^4(\lambda + 2\lambda_1 + \lambda_2 + \alpha + \beta + \theta) \\
&+ s^3(\beta\alpha + \beta\lambda_2 + \alpha\theta + \lambda_2\theta + \beta\theta + 2\lambda_1\alpha + 2\lambda_1\lambda_2 + 2\lambda_1\beta + 2\lambda_1 + \lambda\alpha + \lambda\lambda_2 \\
&+ \beta\lambda + \theta\lambda + \lambda_1\lambda + \lambda_1^2) + s^2(\beta\alpha\theta + \beta\theta\lambda_2 + 2\beta\theta\lambda_1 + 2\beta\lambda_1\lambda_2 + 2\alpha\theta\lambda_1 \\
&+ \beta\alpha\lambda_1 + \theta\lambda_1\lambda_2 + \beta\alpha\lambda + \beta\lambda\lambda_2 + \alpha\lambda\theta + \lambda\theta\lambda_2 + \beta\theta\lambda + \lambda\alpha\lambda_1 + \lambda\lambda_1\lambda_2 + \beta\lambda\lambda_1 \\
&+ \lambda_1\lambda\theta + \lambda_1^2\alpha + \lambda_1^2\lambda_2 + \beta\lambda_1^2 + \lambda_1^2\theta) + s(\beta\alpha\theta\lambda + \beta\lambda\theta\lambda_2 + \beta\lambda\lambda_1\lambda_2 \\
&+ \alpha\theta\lambda\lambda_1 + \beta\lambda\theta\lambda_1 + \beta\alpha\theta\lambda_1 + \beta\theta\lambda_1\lambda_2 + \beta\lambda_1^2\lambda_2 + \alpha\theta\lambda_1^2 + \beta\theta\lambda_1^2) = 0
\end{aligned}$$

In case steady-state availability of the system given by

$$A_0 = \frac{\theta\beta(\alpha\lambda_1 + \lambda_1^2 + \lambda_1\lambda_2 + \lambda\lambda_1)}{\beta\theta(\alpha\lambda + \lambda\lambda_2 + \lambda\lambda_1 + \alpha\lambda_1 + \lambda_1\lambda_2 + \lambda_1^2) + (\beta\lambda_1^2\lambda_2 + \alpha\theta\lambda_1^2 + \beta\lambda\lambda_1\lambda_2 + \alpha\theta\lambda\lambda_1)}$$

From equation (5.2) the time-dependent busy period analysis due to server is given by:

$$B_0^{R*}(t) = \sum_{i=1}^{6} \frac{[\lambda_1\lambda_2(s_i^2 + s_i(\alpha + \lambda_2 + \beta) + \beta\alpha + \beta\lambda_2)]e^{s_i t}}{\prod_{j=1, i\neq j}^{6}(s_i - s_j)}$$

where $s_i (i = 1 \; to \; 6)$ are the roots of the equation

$$\begin{aligned}
&s^6 + s^5(2\alpha + 2\lambda_2 + \beta + \theta + \lambda_1) \\
&+ s^4(2\alpha\beta + 2\beta\lambda_2 + 2\alpha\theta + 2\theta\lambda_2 + \beta\theta + 2\alpha\lambda_1 + 2\lambda_1\lambda_2 + \beta\lambda_1 + \theta\lambda_1 + 3\alpha\lambda_2 \\
&+ \lambda_2^2 + \alpha^2) + s^3(2\alpha\beta\theta + 2\beta\theta\lambda_2 + 2\beta\lambda_1\lambda_2 + 2\alpha\theta\lambda_1 + \beta\theta\lambda_1 + 3\alpha\beta\lambda_2 + \beta\lambda_2^2 \\
&+ 3\alpha\theta\lambda_2 + 3\alpha\lambda_1\lambda_2 + \lambda_1\lambda_2^2 + \theta\lambda_1\lambda_2 + \beta\alpha^2 + \theta\alpha^2 + \alpha^2\lambda_1 + \alpha\beta\lambda_1 + \alpha^2\lambda_2 \\
&+ \alpha\lambda_2^2 + \theta\lambda_2^2) + s^2(\beta\alpha\theta\lambda_2 + \beta\lambda_2^2\theta + \beta\lambda_1\lambda_2^2 + \alpha\theta\lambda_1\lambda_2 + \beta\theta\lambda_1\lambda_2 + \beta\alpha^2\theta \\
&+ \beta\lambda_2\theta\alpha + \beta\lambda_1\lambda_2\alpha + \alpha^2\lambda_1\theta + \beta\lambda_1\theta\alpha + \beta\alpha^2\lambda_2 + \beta\lambda_2^2\alpha + \alpha^2\theta\lambda_2 + \lambda_2^2\alpha\theta \\
&+ \alpha\beta\theta\lambda_2 + \lambda_1\lambda_2\alpha^2 + \lambda_1\lambda_2^2\alpha + \beta\lambda_1\lambda_2\alpha + \lambda_1\theta\lambda_2\alpha) \\
&+ s(\beta\alpha^2\theta\lambda_2 + \beta\lambda_2^2\alpha\theta + \beta\lambda_1\lambda_2^2\alpha + \alpha^2\lambda_1\lambda_2\theta + \beta\lambda_1\lambda_2\alpha\theta) = 0
\end{aligned}$$

In case, Steady-state Busy period analysis due to server is given by

$$B_0^R = \frac{\beta\alpha\lambda_1 + \beta\lambda_1\lambda_2}{\alpha(\alpha\beta\theta + \beta\theta\lambda_2 + \beta\lambda_1\lambda_2 + \alpha\theta\lambda_1 + \beta\theta\lambda_1)}$$

From equation (6.2) the time-dependent busy period due to preventive maintenance of the system is given by:

$$B_0^{P*}(t) = \sum_{i=1}^{6} \frac{[\alpha\lambda_1(s_i^2 + s_i(\theta + \alpha + \lambda_2) + \alpha\theta + \theta\lambda_2)]e^{s_i t}}{\prod_{j=1, j\neq i}^{6}(s_i - s_j)}$$

where $s_i (i = 1 \; to \; 6)$ are the roots of the equation

$$\begin{aligned}
&s^6 + s^5(2\alpha + 2\lambda_2 + \beta + \theta + \lambda_1) \\
&+ s^4(2\alpha\beta + 2\beta\lambda_2 + 2\alpha\theta + 2\theta\lambda_2 + \beta\theta + 2\alpha\lambda_1 + 2\lambda_1\lambda_2 + \beta\lambda_1 + \theta\lambda_1 + 3\alpha\lambda_2 \\
&+ \lambda_2^2 + \alpha^2) + s^3(2\alpha\beta\theta + 2\beta\theta\lambda_2 + 2\beta\lambda_1\lambda_2 + 2\alpha\theta\lambda_1 + \beta\theta\lambda_1 + 3\alpha\beta\lambda_2 + \beta\lambda_2^2 \\
&+ 3\alpha\theta\lambda_2 + 3\alpha\lambda_1\lambda_2 + \lambda_1\lambda_2^2 + \theta\lambda_1\lambda_2 + \beta\alpha^2 + \theta\alpha^2 + \alpha^2\lambda_1 + \alpha\beta\lambda_1 + \alpha^2\lambda_2 \\
&+ \alpha\lambda_2^2 + \theta\lambda_2^2) + s^2(\beta\alpha\theta\lambda_2 + \beta\lambda_2^2\theta + \beta\lambda_1\lambda_2^2 + \alpha\theta\lambda_1\lambda_2 + \beta\theta\lambda_1\lambda_2 + \beta\alpha^2\theta \\
&+ \beta\lambda_2\theta\alpha + \beta\lambda_1\lambda_2\alpha + \alpha^2\lambda_1\theta + \beta\lambda_1\theta\alpha + \beta\alpha^2\lambda_2 + \beta\lambda_2^2\alpha + \alpha^2\theta\lambda_2 + \lambda_2^2\alpha\theta \\
&+ \alpha\beta\theta\lambda_2 + \lambda_1\lambda_2\alpha^2 + \lambda_1\lambda_2^2\alpha + \beta\lambda_1\lambda_2\alpha + \lambda_1\theta\lambda_2\alpha) \\
&+ s(\beta\alpha^2\theta\lambda_2 + \beta\lambda_2^2\alpha\theta + \beta\lambda_1\lambda_2^2\alpha + \alpha^2\lambda_1\lambda_2\theta + \beta\lambda_1\lambda_2\alpha\theta) = 0
\end{aligned}$$

Steady-state busy period analysis due to preventive maintenance is given by

$$B_0^P = \frac{\theta\alpha\lambda_1 + \theta\lambda_1\lambda_2}{\lambda_2(\alpha\beta\theta + \beta\theta\lambda_2 + \beta\lambda_1\lambda_2 + \alpha\theta\lambda_1 + \beta\theta\lambda_1)}$$

The time-dependent expected number of visits can be calculated from the equation (7.2) as

$$N_0^*(t) = \sum_{i=1}^{6} \frac{\{\lambda_1(\lambda_2 + \alpha)[(s_i + \theta)(s_i + \beta)(s_i + \alpha + \lambda_2)]\}e^{s_i t}}{\prod_{j=1, j\neq i}^{6}(s_i - s_j)}$$

where $s_i (i = 1 \; to \; 6)$ are the roots of the equation

$$\begin{aligned}
&s^6 + s^5(2\alpha + 2\lambda_2 + \beta + \theta + \lambda_1) \\
&+ s^4(2\alpha\beta + 2\beta\lambda_2 + 2\alpha\theta + 2\theta\lambda_2 + \beta\theta + 2\alpha\lambda_1 + 2\lambda_1\lambda_2 + \beta\lambda_1 + \theta\lambda_1 + 3\alpha\lambda_2 \\
&+ \lambda_2^2 + \alpha^2) + s^3(2\alpha\beta\theta + 2\beta\theta\lambda_2 + 2\beta\lambda_1\lambda_2 + 2\alpha\theta\lambda_1 + \beta\theta\lambda_1 + 3\alpha\beta\lambda_2 + \beta\lambda_2^2 \\
&+ 3\alpha\theta\lambda_2 + 3\alpha\lambda_1\lambda_2 + \lambda_1\lambda_2^2 + \theta\lambda_1\lambda_2 + \beta\alpha^2 + \theta\alpha^2 + \alpha^2\lambda_1 + \alpha\beta\lambda_1 + \alpha^2\lambda_2 \\
&+ \alpha\lambda_2^2 + \theta\lambda_2^2) + s^2(\beta\alpha\theta\lambda_2 + \beta\lambda_2^2\theta + \beta\lambda_1\lambda_2^2 + \alpha\theta\lambda_1\lambda_2 + \beta\theta\lambda_1\lambda_2 + \beta\alpha^2\theta \\
&+ \beta\lambda_2\theta\alpha + \beta\lambda_1\lambda_2\alpha + \alpha^2\lambda_1\theta + \beta\lambda_1\theta\alpha + \beta\alpha^2\lambda_2 + \beta\lambda_2^2\alpha + \alpha^2\theta\lambda_2 + \lambda_2^2\alpha\theta \\
&+ \alpha\beta\theta\lambda_2 + \lambda_1\lambda_2\alpha^2 + \lambda_1\lambda_2^2\alpha + \beta\lambda_1\lambda_2\alpha + \lambda_1\theta\lambda_2\alpha) \\
&+ s(\beta\alpha^2\theta\lambda_2 + \beta\lambda_2^2\alpha\theta + \beta\lambda_1\lambda_2^2\alpha + \alpha^2\lambda_1\lambda_2\theta + \beta\lambda_1\lambda_2\alpha\theta) = 0
\end{aligned}$$

The steady-state expected no of visit is given by:

$$N_0 = \frac{\theta\beta\lambda_1(\alpha+\lambda_2)^2}{\alpha\lambda_2(\alpha\beta\theta+\beta\theta\lambda_2+\beta\lambda_1\lambda_2+\alpha\theta\lambda_1+\beta\theta\lambda_1)}$$

10.Conclusion

The tabular behaviour of mean time to system failure (MTSF) with respect to maximum rate of operation time (α) is shown in table 2. It is observed that MTSF decrease with the increase of α. And, there is a further decline in their values when direct failure rate (λ) and partial failure rate (λ_1) increase. Tables 3 and 4 reflect respectively availability and profit of the system model decrease with the increase of maximum rate of operation (α), direct failure rate(λ) and partial failure rate (λ_1) for fixed values of other parameters. However, there is a substantial positive change in their values when repair rate (Θ) and preventive maintenance rate (β) increase. On the basis of the results obtained for a particular case it is analyzed that a system which undergoes preventive maintenance after a maximum operation time at partial failure stage can be made more profitable by increasing the repair rate of the system at its complete failure.

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Table 2.

α ↓	Mean Time to System Failure(MTSF)		
	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=2.7	λ=.16,λ_1=.17, λ_2=.21,θ=2.1, β=2.7	λ=.13,λ_1=.20, λ_2=.21,θ=2.1, β=2.7
5	3.550864	3.228058	3.262956
10	3.444336	3.131214	3.149022
15	3.407846	3.098042	3.109995
20	3.389411	3.081283	3.090279
25	3.378289	3.071172	3.078384
30	3.370849	3.064408	3.070426
35	3.365521	3.059565	3.064729
40	3.361519	3.055926	3.060448
45	3.358402	3.053092	3.057114
50	3.355905	3.050823	3.054444

Table 3.

α ↓	Availability				
	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=2.7	λ=.16,λ_1=.17, λ_2=.21,θ=2.1, β=2.7	λ=.13,λ_1=.20, λ_2=.21,θ=2.1, β=2.7	λ=.13,λ_1=.17, λ_2=.21,θ=2.6, β=2.7	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=3.7
5	0.891564	0.880701	0.883569	0.901317	0.904315
10	0.890324	0.879323	0.881995	0.899959	0.903512
15	0.889891	0.878842	0.881444	0.899485	0.903231
20	0.889671	0.878597	0.881163	0.899244	0.903088
25	0.889537	0.878449	0.880992	0.899098	0.903001
30	0.889448	0.878349	0.880878	0.899	0.902943
35	0.889383	0.878278	0.880796	0.898929	0.902902
40	0.889335	0.878224	0.880735	0.898876	0.90287
45	0.889297	0.878183	0.880687	0.898835	0.902846
50	0.889267	0.878149	0.880648	0.898802	0.902826

Table. 4

α ↓	Profit				
	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=2.7,K_1=5000, K_2=150,K_3=75, K_4=50	λ=.16,λ_1=.17, λ_2=.21,θ=2.1, β=2.7,K_1=5000, K_2=150,K_3=75, K_4=50	λ=.13,λ_1=.20, λ_2=.21,θ=2.1, β=2.7,K_1=5000, K_2=150,K_3=75, K_4=50	λ=.13,λ_1=.17, λ_2=.21,θ=2.6, β=2.7,K_1=5000, K_2=150,K_3=75, K_4=50	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=3.7,K_1=5000, K_2=150,K_3=75, K_4=50
5	4432.517	4375.405	4390.878	4482.644	4496.983
10	4426.083	4368.243	4382.708	4475.604	4492.754
15	4423.836	4365.743	4379.847	4473.147	4491.276
20	4422.693	4364.471	4378.389	4471.896	4490.524
25	4422	4363.7	4377.505	4471.138	4490.068
30	4421.536	4363.184	4376.912	4470.63	4489.763
35	4421.202	4362.813	4376.486	4470.266	4489.543
40	4420.952	4362.534	4376.166	4469.992	4489.378
45	4420.756	4362.317	4375.917	4469.778	4489.25
50	4420.6	4362.142	4375.717	4469.607	4489.147

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