Eksploatacja i Niezawodnosc – Maintenance and Reliability
Volume 25 (2023), Issue 1
journal homepage: http://www.ein.org.pl

Article citation info:
Jia S, Yan C, Kang J, Xie H, Wei Y. Optimal allocation of reliability improvement target based on multiple correlation failures and risk uncertainty. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2023: 25(1) http://doi.org/10.17531/ein.2023.1.8

Optimal allocation of reliability improvement target based on multiple correlation failures and risk uncertainty

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Highlights

- a risk assessment machinery is proposed under probability measure,
- the generalized risk models are established based on the cooperative game theory,
- a multi-objective optimal allocation model is presented for reliability improvement,
- the proposed method is more reasonable, complete, and practical.

Abstract

Optimal allocation of the reliability improvement target is essential for the system optimization design. In order to solve the problems that the optimization model is with loss of generality and the validity of the optimal solution is weakened, an optimal allocation method is proposed by considering multiple correlation failures and risk uncertainty in this paper. Two new concepts are presented, such as independent failure results in basic risk, and correlation failure leads to disturbance risk. A risk assessment machinery of "actual risk = basic risk + disturbance risk" is proposed. The action mechanisms of the three correlation failures are studied based on the cooperation game theory, and the generalized risk models are given under probability measure. Considering the improvement cost, the expectation and the variance of the reduction of system risk, a multi-objective optimal allocation model is developed, which is solved by using the PSO algorithm. Finally, the proposed optimal allocation is implemented at the 2-stage NGW planetary reducer, and the results show that it is more efficient and feasible for engineering practice.

Keywords
Multi-objective optimal allocation, reliability improvement, correlation failure, risk uncertainty, probability measure, cooperation game theory, PSO algorithm.

1. Introduction

With the increase in systematization and sophistication of the industrial product, more and more attentions are drawn to the system reliability design. Reliability allocation is the core of the system reliability design process and composed of two parts [27]: (1) the allocation of the system reliability design requirement [13, 17, 31] and (2) optimal allocation of the system reliability improvement target. During the design process, results of the allocation are used to predict system reliability. If the prediction result is less than the system reliability design requirement, optimal allocation of the system reliability improvement target is implemented in further. Hence, the optimal allocation is considered a special or advanced topic that deserves in deep study.

Initially, the optimal allocation is performed based on allocation weights. Yadav and Zhuang [32] proposed to allocate the system reliability improvement target to the subsystems. Cao et.al [2, 3] developed an allocation weight calculation theory by improving and completing the Risk Priority Number (RPN) model. Chen et.al [4] improved the efficiency of obtaining weights via multi-criteria decision-making methods. Zhang et.al [35] and Fiondella et.al [11] introduced the failure correlation factor into the formula of the allocation weight.

A positive weight is assigned to every subsystem through the weight method, but improvements for some subsystems are unworthy or prohibitive under the cost limitation. Hence, Kim and Zuo [18] allocated the system reliability improvement target to priority subsystems based on the optimization model.

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Maryam and Mahdi [24] ranked the priority of subsystems by assessing the feasibility and difficulty of the reliability improvement. Aiming at achieving the system reliability target subject to the cost limitation, Si et.al [28] and Johnston et.al [14] built the reliability optimal allocation model. Kanagaraj et.al [15] and Lai et.al [21] investigated the problem of minimizing the optimization design cost under reliability constraints. Samanta and Basu [26] recognized the nonlinear phenomenon of reliability and gave the growth effort function of reliability. Dai et.al [7] constructed an allocation model considering the common cause failures.

The potential relationship between some objectives and constraints is ignored in the single-objective optimization, such as the approximate positive correlation between reliability and improvement cost. Hence, a multi-objective optimal allocation model is developed, which is generally more complicated and solved by the intelligent optimization algorithm. Duan et.al [9] investigated an optimization allocation method for maximizing the system's theoretical production rate and minimizing the system state entropy. Considering that the reliability, cost, and manufacturing consistency are coupled, Liu et.al [22] put forward a reliability allocation method based on multidisciplinary design optimization. Abdelkader et.al [1] and Kim et.al [16] studied the multi-objective optimal allocation of reliability in series or parallel systems. Liu [23] et.al took the warranty servicing cost as an additional objective for the optimization of the repairable system. Kumar et.al [20] and Zhang et.al [34] solved the multi-objective optimal allocation model by the gray wolf optimizer algorithm and the Particle Swarm Optimization (PSO) algorithm, respectively. Taking the risk as the losses from failures, Todinov et.al [29] extended the concept of the total cost as the sum of the risk and the resources invested in reliability improvement.

Fundamentally speaking, reliability allocation is a kind of predictive design technique [6]. In previous research works, the system attribute was evaluated determinately, and the uncertainty of some variables was ignored in the optimal allocation model, which makes the values of indexes contingent and weakens the validity of the solution. In addition, the conventional models were built at the subsystem or component level and under counting measure, therefore they would lose the generality. In this paper, the multiple correlation failures and the risk uncertainty are studied under probability measure, so that a more complete and reasonable optimal allocation model is proposed for reliability improvement. The proposed model is solved based on the PSO algorithm, and its solution is proved to be more effective and refined.

The rest of this article is organized as follows: the conventional method is reviewed briefly in section 2. In section 3, the correlation failures are investigated based on the cooperative game theory, and the classical RPN model is extended to the probability measure. Simultaneously, the criteria are given to evaluate the system improvement profit, and a multi-objective optimal allocation model is built and solved by the PSO algorithm. In section 4, taking the 2-stage NGW planetary reducer as an example, the efficiency and feasibility of the proposed model are analyzed and discussed. In section 5, some conclusions are drawn.

2. Conventional optimal model

Considering that a system is comprised of m independent components in series, component i includes N_i independent failure modes. Let λ_i be the failure rate of component i during the normal life phase of the bathtub curve under the independent failure. Assuming that the product lifespan obeys the exponential distribution during this period [10, 18], the failure probability F_i of component i can be expressed as:

$$F_i = 1 - \exp(-\lambda_i \cdot t), i = 1, \dots, m. \tag{1}$$

The system failure probability F is shown as:

$$F = 1 - \prod_{i=1}^{m}(1 - F_i). \tag{2}$$

Let λ be the system failure rate under the independent failure, which can be expressed as,

$$\lambda = \sum_{i=1}^{m} \lambda_i. \tag{3}$$

Now, make a statement that the variables with or without superscript "*" symbolize the state before or after improvement, respectively. Hence, Eq. (4) holds for this series system.

$$\lambda - \lambda^* = \sum_{i=1}^{m}(\lambda_i - \lambda_i^*). \tag{4}$$

Because the failure rate is an important measurement of product reliability [30], the difference of $\lambda - \lambda^*$ is taken usually as the improvement target during the system reliability optimization. Naming $\Delta = \lambda - \lambda^*$ as the reduction of the system failure rate, Δ is allocated down based on an optimization model in the optimal allocation of the system reliability improvement target.

Minimizing the system improvement cost is common in the optimal allocation, however it is subject to reliability constraints [21]. Recently, the risk is taken as a criterion for judging the system quality, and minimizing the system risk is taken as an objective of the system reliability improvement. Hence, maximizing the reduction of system risk is also taken into account in the optimal allocation model [18].

Let r_s denote the system failure risk under independent failure, which is quantified as Eq. (5) based on the RPN model [33].

$$r_s = \lambda \cdot s, \tag{5}$$

where, s denotes the system failure severity under independent failure.

In the same manner, the component failure risk r_i and failure mode risk r_{ij} under independent failure are as Eq. (6) and Eq. (7).

$$r_i = \lambda_i \cdot s_i, \tag{6}$$

where, s_i is the failure severity of component i under independent failure.

$$r_{ij} = \lambda_{ij} \cdot s_{ij}, \tag{7}$$

where, λ_{ij} and s_{ij} represent the occurrence rate and severity of failure mode j of component i under independent failure, respectively.

Considering the risk accumulation effect in the series system, the system risk under independent failure is calculated by Eq. (8).

$$\lambda \cdot s = \sum_{i=1}^{m}(\lambda_i \cdot s_i) = \sum_{i=1}^{m}\sum_{j=1}^{N_i}(\lambda_{ij} \cdot s_{ij}) \tag{8}$$

By adjusting Eq. (8), s and s_i are obtained as Eq. (9) and Eq. (10), respectively.

$$s = \sum_{i=1}^{m}(\frac{\lambda_i}{\lambda} \cdot s_i), \tag{9}$$

$$s_i = \sum_{j=1}^{N_i}(\frac{\lambda_{ij}}{\lambda_i} \cdot s_{ij}). \tag{10}$$

Assuming $s_i = s_i^*$ in the conventional methods [16, 18], the optimal allocation model is shown as Eq. (11). In Eq. (11), the objective function includes the cost of system improvement and the reduction of system risk.

$$\min \quad \sum_{i=1}^{m}\left\{\delta_i \cdot \ln\left(\frac{\lambda_i}{\lambda_i^*}\right)\right\} - \sum_{i=1}^{m}\{(\lambda_i - \lambda_i^*)\cdot s_i\}, \tag{11}$$

$$\text{s.t} \quad \begin{cases} 0<\lambda_i^*\leq\lambda_i \\ \sum_{i=1}^{m}(\lambda_i - \lambda_i^*) = \Delta \end{cases},$$

where δ_i is the difficulty coefficient for improving component *i*.

In the conventional method, Δ is accomplished by decreasing λ_i to λ_i^*, however the reduction of $\lambda_i - \lambda_i^*$ achieved fundamentally by decreasing λ_{ij} to λ_{ij}^* is not taken into consideration. Hence, an analytic hierarchy model of "system–component–failure mode" should be considered before analyzing the conventional model.

(1) λ_i is the sum of several λ_{ij} , and a determinate reduction of $\lambda_i - \lambda_i^*$ can be achieved by many schemes. These schemes determine which λ_{ij} should be decreased and how many. The improvement difficulties and costs of diverse failure modes may be different, so complexities and costs of different schemes are also different actually. That is to say that the conventional formula of the component improvement cost in Eq. (11) is with loss of generality.

(2) s_i is a function subject to several λ_{ij} in Eq. (10). Once the system reliability is improved and fundamentally λ_{ij} is decreased to λ_{ij}^*, s_i^* is likely to be different from s_i. Hence, the formula of the component risk reduction in Eq. (11) may not work actually, and the validity of the solution of the conventional model is doubtful.

(3) The law of conservation of energy holds for a system, so the loads for most components are mutually correlative [27]. From the perspective of failure mechanism, a component failure may cause or be caused by another component failure. From the risk point of view, the component failure risk may be influenced by the interaction among various components. Furthermore, the component failure stems from failure mode occurring, so the stress corresponding to the failure mode is a micro reflection of component load. Hence, it can be further derived that various failure modes can affect each other and the failure mode risk may be influenced by the interaction. Here, the various failure modes may belong to the same or different components.

If this interaction is taken as a property of correlation, it would be indefinite during the system reliability design process. Hence, considering the correlation during the reliability allocation, the adverse events within the system are uncertain. Simultaneously, the uncertainty of correlation would deepen the uncertainty of the actual risk in further. Therefore, the conventional risk model can only describe the determinate risk of independent event.

Above issues are not taken into consideration comprehensively in conventional optimal allocation model, which would weaken the validity of its solution. Hence, considering the multiple correlation failures and risk uncertainty, an optimal allocation is proposed in this study.

3. Proposed optimal allocation

3.1 Risk model under probability measure

Considering that a system is comprised of m components in series, the component i includes N_i failure modes. The improvement target Δ is accomplished by decreasing λ_{ij} to λ_{ij}^*, and the three statements need to be declared ahead of time.

(1) λ_{ij} cannot be decreased to 0 in any case, so the number of failure modes for a component will keep constant after improvement.

(2) s_{ij} refers to the disruption to system mission resulting from a failure mode, which doesn't depend on whether the failure mode occurs or not [8]. Hence, $s_{ij} = s_{ij}^*$ holds in a definite system [18].

(3) Let R_s, R_i and R_{ij} denote the risk of system, component i and failure mode j of component i under the uncertain correlation, respectively. In fact, they are regarded as the actual risks in this study.

In the series system, the components and failure modes are countable. Under the action of uncertain correlation, the components failure or failure modes occurrence are not mutually exclusive, but the failure events related to components or failure modes are enumerable. Therefore, elementary events in the sample space of system failure are with randomness, and both R_i and R_{ij} are the random variables. Next, the generalized system risk model is derived from the risk accumulation effect under probability measure.

The independent failure can be regarded as a special case of the correlation failure with zero interaction. Either all components failure or all failure modes occurrence are mutually exclusive under the independent failure assumption. Therefore, based on these premises, the correlation has no impact on risks of independent failure events and r_{ij}, r_i and r_s are determinate. Regarding the independent failure event as the basic event arousing risk, the system basic risk is formulated as Eq. (12) based on the risk accumulation effect [18].

$$r_s = \sum_{i=1}^{m} r_i = \sum_{i=1}^{m}\sum_{j=1}^{N_i} r_{ij}. \tag{12}$$

The nonzero interaction results in the possible fluctuation of actual risk around basic risk, and this finite fluctuation is defined as disturbance risk. Hence, a risk assessment machinery of "actual risk = basic risk + disturbance risk" under correlation failure is proposed. Here, both disturbance risk and actual risk are random variables.

Because the basic risk is a special actual risk, Eq. (12) demonstrates not only the relationship of basic risks, but also the relationship of actual risks to some extent. Therefore, extending Eq. (12) to probability domain, R_s is a multidimensional random variable with respect to R_i or R_{ij}. The system actual risk model under probability measure is given as Eq. (13).

$$R_s = \sum_{i=1}^{m} R_i = \sum_{i=1}^{m}\sum_{j=1}^{N_i} R_{ij}. \tag{13}$$

Subtracting basic risk from the actual risk, the system disturbance risk is also a multidimensional random variable with respect to disturbance risks of components or failure modes. Therefore, the system disturbance risk is the sum of all disturbance risks of failure modes. The disturbance risk model of system under probability measure can be obtained as Eq. (14).

$$\varepsilon_{rs} = \sum_{i=1}^{m} \varepsilon_{ri} = \sum_{i=1}^{m}\sum_{j=1}^{N_i} \varepsilon_{rij}, \tag{14}$$

where, ε_{rs}, ε_{ri} and ε_{rij} denote the disturbance risk of system, component i and failure mode j of component i, respectively.

r_{ij} is a function with respect to λ_{ij} and s_{ij} in Eq. (7). ε_{rij} mirrors a risk fluctuation caused by correlation failure, of which the dominant factors are the action effects of correlation failure on λ_{ij} and s_{ij}. Furthermore, correlation failure uncertainty results in the uncertainty of ε_{rij}, of which the intrinsic cause is that action effects of uncertain correlation failure on λ_{ij} and s_{ij} are uncertain. The demonstration is given in detail in next section.

The correlation failure is usually summarized as cascading failure, common cause failure, and negative correlation failure [2]. The cascading failure refers to the behavior that a component failure facilitates subsequently the failure of another component in a system, which has an effect of magnifying λ_i of the latter and s_i of the former compared with the independent failure. Considering that the interaction is reversible, cascading failure would increase λ_i and s_i of both involved components. The common cause failure means that several components in a system fail synchronously due to external causes. It is a special failure manner of components, and amplifies λ_i of these components compared with independent failure. The negative correlation failure is that one component failure may make another component tend to be without failure. Hence, it has an effect of decreasing λ_i of the latter and s_i of the former compared with the independent failure. Considering the reversibility of the interaction, negative correlation failure would decrease λ_i and s_i of both involved components.

Because the deep reason of component failure is the occurrence of failure modes, the effects of correlation failure on the component can be effectively transformed into the effects on the failure mode. The details are shown in Table 1. In Table 1, “Inc.”, “Dec.” and “NG” are the abbreviations of “Increase”, “Decrease”, and “Not Given”, respectively. “Inc.” or “Dec.” means that the correlation failure would increase or decrease the value of the risk factor, respectively. “NG” means that the correlation failure has no impact on the value of the risk factor.

Table 1. Impact of correlation failure on failure mode risk factors

Correlation failure type	Effect on failure mode risk factors		
	On λ_{ij}	On s_{ij}	On r_{ij}
Cascading failure	Inc.	Inc.	Inc.
Common cause failure	Inc.	NG	Inc.
Negative correlation failure	Dec.	Dec.	Dec.

The three kinds of correlation failure can be uniformly regarded as joint failure [3], and the uncertainty of its internal interaction dominates the uncertainty of correlation. In the light of this, the cooperative game theory is brought to demonstrate the action mechanism of correlation failure, and alliances of failure modes are taken as the game players.

In the system, there are totally $\sum_{i=1}^{m} N_i = n$ failure modes and $\binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n} = (2^n - n - 1)$ alliances of failure modes. Because the population of alliances is relatively large, it is supposed that every alliance is of the same occurrence chance from the scientific research and engineering practice point of view and simplifying the question. Simultaneously, the occurrence of every alliance has the same consequences for the system, which makes the system fail. Therefore, a standard of classification is given that the alliance type is dependent on the quantity of its failure modes, and the probability of a type of alliances including specific failure mode j and k-1 other failure modes is calculated as $\frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}$. Next, the action effect of this type of alliances on its internal failure mode j is quantified based on Table 1.

Under the cascading failure, interaction among the failure modes is reversible, therefore every other failure mode in the alliance has a positive action on failure mode j. The schematic diagram of cascading failure is shown as Fig. 1.

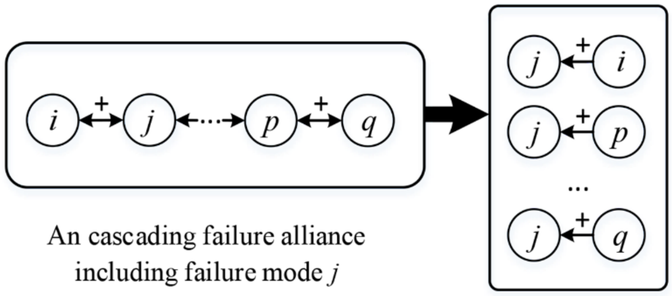


Fig. 1. Schematic diagram of cascading failure

In this study, the action of every other failure mode on failure mode j is defined as one meta-action, and it is considered that the meta-actions can be linearly accumulated. Hence, for a cascading failure alliance including the specific failure mode j and k-1 other failure modes, it has k-1 positive meta-actions on both λ_{ij} and s_{ij}. Without loss of generality, for a cascading failure alliance with k failure modes, the distribution law of Action Value of Alliance (AVA) on λ_{ij} and s_{ij} is formulated as Eq. (15).

$$\Pr(\text{AVA} = k-1) = \frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}, k = 2, \ldots, n \quad (15)$$

Under common cause failure, occurrence of an alliance means that all of the minor alliances constituted of its internal failure modes occur synchronously. Every minor alliance can be broken down into several minimal alliances with 2 failure modes. Therefore, for an alliance including specific failure mode j and k-1 other failure modes, the occurrence of it can be replaced by the fact that k-1 minimal alliances containing failure mode j occur synchronously. The schematic diagram of common cause failure is shown as Fig. 2.

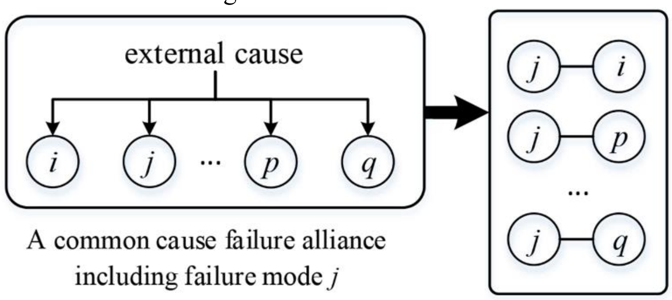


Fig. 2. Schematic diagram of common cause failure

Every minimal alliance containing failure mode j has a basic amplification effect on λ_{ij}, and it can be equivalent to a phenomenon that the other failure mode in every minimal alliance has a positive action on failure mode j. According to the definition of the meta-action, it can be derived that every minimal alliance containing failure mode j has a positive meta-action on λ_{ij}. Hence, for a common cause failure alliance including the specific failure mode j and k-1 other failure modes, it has k-1 positive meta-actions on λ_{ij}. Without loss of generality, for a common cause failure alliance with k failure modes, the distribution law of AVA on λ_{ij} is also formulated as Eq. (15).

Under negative correlation failure, interaction among the failure modes is reversible, therefore every other failure mode has a negative action on failure mode j in an alliance. The schematic diagram of negative correlation failure is shown as Fig. 3.

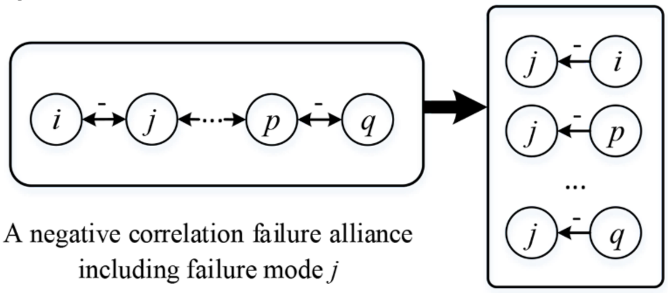


Fig. 3. Schematic diagram of negative correlation failure

It is supposed that the action of every failure mode is equivalent and can be linearly accumulated. For a negative correlation failure alliance including the specific failure mode j and k-1 other failure modes, it has k-1 negative meta-actions on both λ_{ij} and s_{ij}. Without loss of generality, for a negative correlation failure alliance with k failure modes, the distribution law of AVA on both λ_{ij} and s_{ij} is formulated as Eq. (16).

$$\Pr(\text{AVA} = 1-k) = \frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}, k = 2, \dots, n \tag{16}$$

Based on the classical model of probability and simplifying the question, it is supposed that three kinds of correlation failures for an undefined system have the equal occurrence probabilities. Considering synthetically three kinds of correlation failures, the expectation and variance of AVA on λ_{ij} are calculated as following, respectively.

$$E(\varepsilon_o) = \sum_{k=2}^{n}\left\{\left(\frac{(k-1)+(k-1)+(1-k)}{3}\right)\cdot\frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}\right\} > 0,$$

$$D(\varepsilon_o) = \sum_{k=2}^{n}\left\{\left(\frac{2\cdot(k-1-E(\varepsilon_o))^2+(1-k-E(\varepsilon_o))^2}{3}\right)\cdot\frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}\right\} > 0,$$

where, ε_o denotes AVA on λ_{ij}. $E(\cdot)$ and $D(\cdot)$ symbolize the variable expectation and variance, respectively.

The expectation and variance of AVA on s_{ij} are calculated as following, respectively.

$$E(\varepsilon_s) = \sum_{k=2}^{n}\left\{\left(\frac{(k-1)+(1-k)}{3}\right)\cdot\frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}\right\} = 0,$$

$$D(\varepsilon_s) = \sum_{k=2}^{n}\left\{\left(\frac{(k-1-E(\varepsilon_s))^2+(0-E(\varepsilon_s))^2+(1-k-E(\varepsilon_s))^2}{3}\right)\cdot\frac{(n-1)!}{(k-1)!\cdot(n-k)!\cdot(2^{n-1}-1)}\right\} > 0,$$

where, ε_s denotes AVA on s_{ij}.

Hence, the actual risks of failure mode before and after improvement are given as Eq. (17) and Eq. (18), respectively.

$$R_{ij} = (\lambda_{ij} + \varepsilon_o)\cdot(s_{ij} + \varepsilon_s), \tag{17}$$

$$R_{ij}^* = (\lambda_{ij}^* + \varepsilon_o)\cdot(s_{ij} + \varepsilon_s). \tag{18}$$

The disturbance risks of failure mode before and after improvement are also given as Eq. (19) and Eq. (20), respectively.

$$\varepsilon_{rij} = \lambda_{ij}\cdot\varepsilon_s + \varepsilon_o\cdot s_{ij} + \varepsilon_o\cdot\varepsilon_s, \tag{19}$$

$$\varepsilon_{rij}^* = \lambda_{ij}^*\cdot\varepsilon_s + \varepsilon_o\cdot s_{ij} + \varepsilon_o\cdot\varepsilon_s. \tag{20}$$

3.2 Evaluation of system improvement revenue

R_s is a random variable, and $E(R_s)$ reflects an average influence of all the potential adverse events on system. Hence, the first measure of system improvement revenue $I_1(\lambda,\lambda^*)$ is proposed as Eq. (21).

$$I_1(\lambda,\lambda^*) = E(R_s) - E(R_s^*) \tag{21}$$

In addition, $D(R_s)$ is the overall degree of deviation for all the possible values of R_s from $E(R_s)$. The larger $D(R_s)$ is, the more dispersive R_s is. If the possible risk values of a system vary widely enough, the system would be uncontrollable. From the perspective of controlling ability, a large risk variance would lead to a terrible system risk. Therefore, the second measure of the system improvement revenue $I_2(\lambda,\lambda^*)$ is proposed as Eq. (22).

$$I_2(\lambda,\lambda^*) = D(R_s) - D(R_s^*) \tag{22}$$

In Eq. (17) and Eq. (18), both λ_{ij} and s_{ij} are numerical variables, which are obtained by taking an exponential transformation of the ten-point linear scale and have definite dimensions. On the contrary, ε_o and ε_s are random variables and in meta-action, which is a contrived measurement scale of the action effect of correlation failure. Therefore, addition of the two kinds of variables have only formal meaning rather than practical meaning. In addition, the practical effects of one meta-action for λ_{ij} and s_{ij} are likely to be different. In order to avoid the dilemma that $E(R_s)$ and $D(R_s)$ are incalculable, the changes of $E(R_s)$ and $D(R_s)$ are replaced with the numerical characteristics of ΔR_s to evaluate the improvement revenue. ΔR_s is given as following.

$$\Delta R_s = R_s - R_s^* = \sum_{i=1}^{m}\sum_{j=1}^{N_i}\{(\lambda_{ij} - \lambda_{ij}^*)\cdot(s_{ij} + \varepsilon_s)\} \tag{23}$$

According to the property of expectation, $E(\Delta R_s)$ can be written as:

$$E(\Delta R_s) = E(R_s) - E(R_s^*). \tag{24}$$

Because $E(\varepsilon_s)$ is equal to 0, $E(\Delta R_s)$ can also be written as

$$E(\Delta R_s) = \sum_{i=1}^{m}\sum_{j=1}^{N_i} E\left((\lambda_{ij} - \lambda_{ij}^*)\cdot s_{ij}\right) + \sum_{i=1}^{m}\sum_{j=1}^{N_i} E\left((\lambda_{ij} - \lambda_{ij}^*)\cdot\varepsilon_s\right) = \sum_{i=1}^{m}\sum_{j=1}^{N_i}\{(\lambda_{ij} - \lambda_{ij}^*)\cdot s_{ij}\}, \tag{25}$$

where, $\lambda_{ij} \geq \lambda_{ij}^* > 0$.

The less λ_{ij}^* is, the larger $E(\Delta R_s)$ is. According to Eq. (24), the larger $E(\Delta R_s)$ is, the more $E(R_s) - E(R_s^*)$ is. Considering the calculability of $E(\Delta R_s)$, the first measure can be adjusted by a proposed criterion that the larger $E(\Delta R_s)$ is, the better the system improvement revenue is.

According to Eq. (23),

$$D(\Delta R_s) = D(R_s) + D(R_s^*) - 2Cov(R_s, R_s^*) \tag{26}$$

$$D(\Delta R_s) = \sum_{\substack{1\leq i,p\leq m\\ 1\leq j,q\leq N_i}} Cov\left((\lambda_{ij} - \lambda_{ij}^*)\cdot\varepsilon_s, (\lambda_{pq} - \lambda_{pq}^*)\cdot\varepsilon_s\right) = \sum_{\substack{1\leq i,p\leq m\\ 1\leq j,q\leq N_i}}\{(\lambda_{ij} - \lambda_{ij}^*)\cdot(\lambda_{pq} - \lambda_{pq}^*)\cdot D(\varepsilon_s)\}, \tag{27}$$

where, $Cov(\cdot)$ symbolizes the covariance between variables.

The less λ_{ij}^* (or λ_{pq}^*) is, the larger $D(\Delta R_s)$ is.

According to Eq. (13), Eq. (17), and Eq. (18),

$$D(R_s^*) = \sum_{\substack{1\leq i,p\leq m\\ 1\leq j,q\leq N_i}} Cov(R_{ij}^*, R_{pq}^*) = \sum_{\substack{1\leq i,p\leq m\\ 1\leq j,q\leq N_i}} Cov(\lambda_{ij}^*\cdot\varepsilon_s + \varepsilon_o\cdot s_{ij} + \varepsilon_o\cdot\varepsilon_s, \lambda_{pq}^*\cdot\varepsilon_s + \varepsilon_o\cdot s_{pq} + \varepsilon_o\cdot\varepsilon_s) \tag{28}$$

The covariance in Eq. (28) can be further decomposed and expressed as the sum of several covariance, where the covariance with respect to λ_{ij}^* (or λ_{pq}^*) are shown as following.

$$Cov(\lambda_{ij}^* \cdot \varepsilon_s, \lambda_{pq}^* \cdot \varepsilon_s) = \lambda_{ij}^* \cdot \lambda_{pq}^* \cdot D(\varepsilon_s) > 0,$$
$$Cov(\lambda_{ij}^* \cdot \varepsilon_s, \varepsilon_o \cdot s_{pq}) = \lambda_{ij}^* \cdot s_{pq} \cdot Cov(\varepsilon_s, \varepsilon_o) > 0,$$
$$Cov(\lambda_{ij}^* \cdot \varepsilon_s, \varepsilon_o \cdot \varepsilon_s) = \lambda_{ij}^* \cdot Cov(\varepsilon_s, \varepsilon_o \cdot \varepsilon_s) = 0,$$
$$Cov(\varepsilon_o \cdot s_{ij}, \lambda_{pq}^* \cdot \varepsilon_s) = s_{ij} \cdot \lambda_{pq}^* \cdot Cov(\varepsilon_o, \varepsilon_s) > 0,$$
$$Cov(\varepsilon_o \cdot \varepsilon_s, \lambda_{pq}^* \cdot \varepsilon_s) = \lambda_{pq}^* \cdot Cov(\varepsilon_o \cdot \varepsilon_s, \varepsilon_s) = 0.$$

According to the above results, it can be derived that the less λ_{ij}^* (or λ_{pq}^*) is, the less $D(R_s^*)$ is. Simultaneously, because $D(R_s)$ is not subject to λ_{ij}^* (or λ_{pq}^*) in Eq. (26), it can be further derived that the less λ_{ij}^* (or λ_{pq}^*) is, the less $Cov(R_s, R_s^*)$ is. Hence, the conclusion is drawn that a larger $D(\Delta R_s)$ means a more reduction of $D(R_s) - D(R_s^*)$. Considering the calculability of $D(\Delta R_s)$, the second measure can be adjusted by another proposed criterion that the larger $D(\Delta R_s)$ is, the better the system improvement revenue is.

3.3 Evaluation of system improvement cost

The series system is improved by decreasing λ_i to λ_i^*, and the cost of system improvement is calculated as the sum of costs of components improvement. The reduction of $\lambda_i - \lambda_i^*$ can be achieved fundamentally by decreasing λ_{ij} to λ_{ij}^*, so the cost of component improvement is the sum of improvement costs of failure modes. Associated with λ_{ij} and $\lambda_{ij} - \lambda_{ij}^*$, the improvement cost of failure mode is formulated as the natural logarithm of $(\lambda_{ij}/\lambda_{ij}^*)$. Hence, the system improvement cost model at the failure mode level is proposed as follow.

$$C(\lambda, \lambda^*) = \sum_{i=1}^{m} \sum_{j=1}^{N_i} \left\{ \delta_{ij} \cdot ln\left(\frac{\lambda_{ij}}{\lambda_{ij}^*}\right) \right\}, \tag{29}$$

where, $C(\cdot)$ symbolizes the improvement cost and δ_{ij} is the difficulty coefficient for improving failure mode *j* of component *i*.

3.4 Optimal allocation model based on PSO algorithm

The optimal allocation of system reliability improvement target is aimed at maximizing the system improvement revenue and minimizing the system improvement cost under some constraints. An advanced optimal allocation model is proposed as follow based on the above demonstration.

$$\min \ \sum_{i=1}^{m} \sum_{j=1}^{N_i} \left\{ \delta_{ij} \cdot ln\left(\frac{\lambda_{ij}}{\lambda_{ij}^*}\right) \right\} - \sum_{i=1}^{m} \sum_{j=1}^{N_i} \left\{ (\lambda_{ij} - \lambda_{ij}^*) \cdot s_{ij} \right\} - \sum_{\substack{1 \le i,p \le m \\ 1 \le j,q \le N_i}} \left\{ (\lambda_{ij} - \lambda_{ij}^*) \cdot (\lambda_{pq} - \lambda_{pq}^*) \cdot D(\varepsilon_s) \right\}, \tag{30}$$

$$\text{s.t} \begin{cases} 0 < \lambda_{ij}^* + \varepsilon_o \le \lambda_{ij} + \varepsilon_o \\ \sum_{i=1}^{m} \sum_{j=1}^{N_i} \{(\lambda_{ij} + \varepsilon_o) - (\lambda_{ij}^* + \varepsilon_o)\} = \Delta \end{cases}$$

The constraints in above model are associated with the random variables ε_o. According to the probability statistical characteristics of ε_o, it is absolutely positively biased based on the "3σ"principle. Simultaneously, compared with λ_{ij} and λ_{ij}^*, the magnitude of ε_o is very small. Hence, within the error rang allowed, the constraints can be simplified via the scaling method as follow.

$$\text{s.t} \quad \begin{cases} 0 < \lambda_{ij}^* \le \lambda_{ij} \\ \sum_{i=1}^{m} \sum_{j=1}^{N_i} \{\lambda_{ij} - \lambda_{ij}^*\} = \Delta \end{cases} \tag{31}$$

Additionally, the constraints in optimal allocation model based on the PSO algorithm must be inequalities. Therefore, Eq. (31) is adjusted as follow [12].

$$\text{s.t} \quad \lambda_{ijmin} \le \lambda_{ij}^* \le \lambda_{ijmax}, \tag{32}$$

where,

$$\begin{cases} \lambda_{ijmin} = max\{0, \sum_{i=1}^{m} \sum_{j=1}^{N_i} \{\lambda - \Delta - (\lambda_{11}^* + \lambda_{12}^* + \cdots + \lambda_{i(j-1)}^*) - (\lambda_{i(j+1)} + \cdots + \lambda_{m(N_m)})\}\} \\ \lambda_{ijmax} = min\{\lambda_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{N_i} \{\lambda - \Delta - (\lambda_{11}^* + \lambda_{12}^* + \cdots + \lambda_{i(j-1)}^*) - 0\}\} \end{cases}$$

The principle of PSO algorithm is a swarm of particles looking for the global optimal solution in the *n*-dimensional space [5]. In the optimal allocation problem of system reliability improvement target, the fitness function is the objective function in Eq. (30), and the occurrence rates of n failure modes constitute the *n*-dimensional space. Thereby, the calculation procedure of PSO algorithm for this optimal problem is shown as Fig.4.

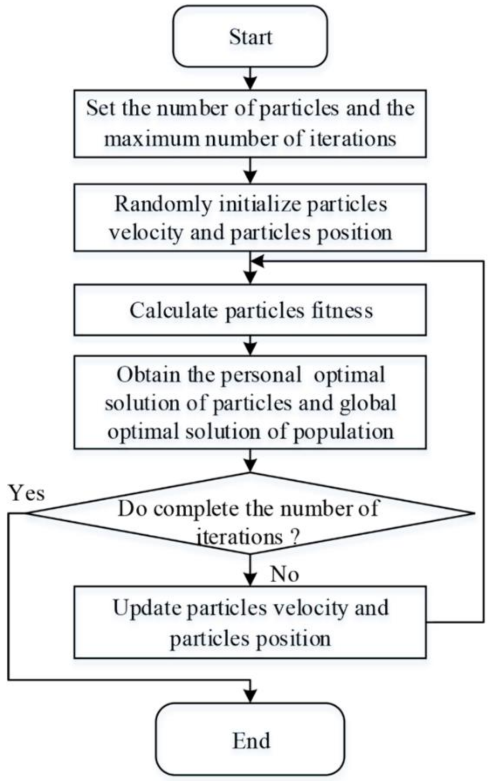


Fig. 4. Procedure of PSO algorithm

4. Numerical example and discussion

4.1 Numerical example

The schematic of 2-stage NGW planetary reducer in the large torque hub drive system is as Fig. 5.

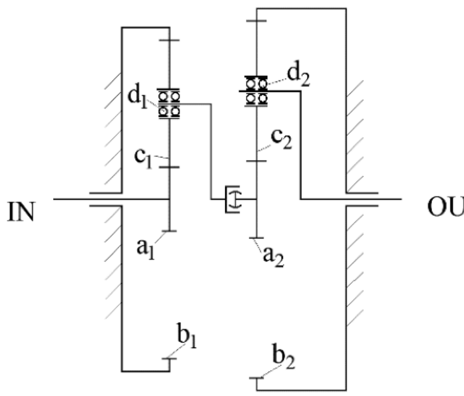


Fig. 5. Schematic of 2-stage NGW planetary reducer: a_1-1st stage sun gear, b_1-1st stage internal gear, c_1-1st stage planet gear, d_1-1st stage bearing, a_2-2nd stage sun gear, b_2-2nd stage internal gear, c_2-2nd stage planet gear, d_2-2nd stage bearing.

In the reference [13], the allocation of system reliability design requirement is implemented for this reducer so that components are configured with λ_i. As a kind of empirical evaluation for λ_{ij}, the occurrence conversion value is largely proportional to λ_{ij}. Hence, λ_i may as well be decomposed into λ_{ij} in the proportion of occurrence conversion value. The full results of the allocation of system reliability design requirement are listed as Table 2.

Table 2. Results of the allocation of system reliability design requirement

	Component	Severity of component	Failure rate		Failure mode	Severity of failure mode	Occurrence rate
i		s_i	λ_i	j		s_{ij}	λ_{ij}
1	1st stage sun gear	601.85	0.000100	1	FM11 Surface pitting	24.53	0.000079
				2	FM12 Surface gluing	270.43	0.000015
				3	FM13 Tooth fracture	601.85	0.000006
2	1st stage internal gear	121.51	0.003807	1	FM 21 Surface pitting	4.95	0.000276
				2	FM 22 Ring fracture	121.51	0.003531
3	1st stage planet gear	403.43	0.000600	1	FM31 Surface pitting	11.02	0.000351
				2	FM 32 Surface gluing	181.27	0.000151
				3	FM 33 Tooth fracture	403.43	0.000098
4	1st stage bearing	270.43	0.030047	1	FM 41 Fatigue exfoliation	11.02	0.010168
				2	FM 42 Wear out	4.95	0.015532
				3	FM 43 gluing	270.43	0.004347
5	2nd stage sun gear	1 363.96	0.000100	1	FM 51 Surface pitting	54.60	0.000093
				2	FM 52 Surface gluing	270.43	0.000003
				3	FM 53 Tooth fracture	1 363.96	0.000004
6	2nd stage internal gear	121.51	0.002102	1	FM 61 Surface pitting	4.95	0.000628
				2	FM 62 Ring fracture	121.51	0.001474
7	2nd stage planet gear	403.43	0.000300	1	FM 71 Surface pitting	11.02	0.000220
				2	FM 72 Surface gluing	181.27	0.000040
				3	FM 73 Tooth fracture	403.43	0.000040
8	2nd stage bearing	270.43	0.037494	1	FM 81 Fatigue exfoliation	11.02	0.008658
				2	FM 82 Wear out	4.95	0.020178
				3	FM 83 Gluing	270.43	0.008658
	total		0.074550		total		0.074550

During the optimal allocation of system reliability improvement target, referring to the mechanical failure distribution [18], λ_i^* and λ_i^{**} denote the component failure rates at the stable failure stage after improvement when $\Delta = 0.003630$ and $\Delta = 0.006151$ [18], respectively. Simultaneously, λ_{ij}^* and λ_{ij}^{**} denote the failure mode occurrence rates after improvement based on the above same condition.

According to the conventional model, λ_i^* and λ_i^{**} are allocated to components based on Eq. (11). Because the assumption $s_i = s_i^*$ means $\lambda_{ij}/\lambda_i = \lambda_{ij}^*/\lambda_i^*$ for the component i, the full results of the conventional optimal allocation are shown in Table 3. It is generally believed that allocation results are sensitive to the difficulty coefficient, so λ_{ij}^* and λ_{ij}^{**} with respect to different δ_i are listed in Tables 3(a), (b), (c) and (d), respectively.

Table 3. Results of conventional improvement target optimal allocation

(a)

Component	Difficulty coefficient	Failure rate		Failure mode	Occurrence rate	
i	δ_i	λ_i^*	λ_i^{**}		λ_{ij}^*	λ_{ij}^{**}
1	9	0.000100	0.000100	FM11	0.000079	0.000079
				FM12	0.000015	0.000015
				FM13	0.000006	0.000006
2	9	0.003807	0.003807	FM 21	0.000276	0.000276
				FM 22	0.003531	0.003531
3	9	0.000600	0.000600	FM31	0.000351	0.000351
				FM 32	0.000151	0.000151
				FM 33	0.000098	0.000098
4	9	0.030047	0.030047	FM 41	0.010168	0.010168
				FM 42	0.015532	0.015532
				FM 43	0.004347	0.004347
5	9	0.000100	0.000100	FM 51	0.000093	0.000093
				FM 52	0.000003	0.000003
				FM 53	0.000004	0.000004
6	9	0.002102	0.002102	FM 61	0.000628	0.000628
				FM 62	0.001474	0.001474
7	9	0.000300	0.000300	FM 71	0.000220	0.000220
				FM 72	0.000040	0.000040
				FM 73	0.000040	0.000040
8	9	**0.033864**	**0.031343**	FM 81	**0.007820**	**0.007238**
				FM 82	**0.018224**	**0.016868**
				FM 83	**0.007820**	**0.007237**
total		**0.070920**	**0.068399**		**0.070920**	**0.068399**

(b)

Component	Difficulty coefficient	Failure rate		Failure mode	Occurrence rate	
i	δ_i	λ_i^*	λ_i^{**}		λ_{ij}^*	λ_{ij}^{**}
1	**5**	0.000100	0.000100	FM11	0.000079	0.000079
				FM12	0.000015	0.000015
				FM13	0.000006	0.000006
2	**6**	0.003807	0.003807	FM 21	0.000276	0.000276
				FM 22	0.003531	0.003531
3	**2**	0.000600	0.000600	FM31	0.000351	0.000351
				FM 32	0.000151	0.000151
				FM 33	0.000098	0.000098
4	9	0.030047	0.030047	FM 41	0.010168	0.010168
				FM 42	0.015532	0.015532
				FM 43	0.004347	0.004347
5	**4**	0.000100	0.000100	FM 51	0.000093	0.000093
				FM 52	0.000003	0.000003
				FM 53	0.000004	0.000004
6	**7**	0.002102	0.002102	FM 61	0.000628	0.000628
				FM 62	0.001474	0.001474
7	**1**	0.000300	0.000300	FM 71	0.000220	0.000220
				FM 72	0.000040	0.000040
				FM 73	0.000040	0.000040
8	9	**0.033864**	**0.031343**	FM 81	**0.007820**	**0.007238**
				FM 82	**0.018224**	**0.016868**
				FM 83	**0.007820**	**0.007237**
		0.070920	**0.068399**		**0.070921**	**0.068399**

(c)

Component	Difficulty coefficient	Failure rate		Failure mode	Occurrence rate	
i	δ_i	λ_i^*	λ_i^{**}		λ_{ij}^*	λ_{ij}^{**}
1	5	0.000100	0.000100	FM 11	0.000079	0.000079
				FM 12	0.000015	0.000015
				FM 13	0.000006	0.000006
2	6	0.003807	0.003807	FM 21	0.000276	0.000276
				FM 22	0.003531	0.003531
3	**3**	0.000600	0.000600	FM 31	0.000351	0.000351
				FM 32	0.000151	0.000151
				FM 33	0.000098	0.000098
4	**8**	0.030047	**0.028889**	FM 41	0.010168	**0.009776**
				FM 42	0.015532	**0.014933**
				FM 43	0.004347	**0.004180**
5	4	0.000100	0.000100	FM 51	0.000093	0.000093
				FM 52	0.000003	0.000003
				FM 53	0.000004	0.000004
6	7	0.002102	0.002102	FM 61	0.000628	0.000628
				FM 62	0.001474	0.001474
7	1	0.000300	0.000300	FM 71	0.000220	0.000220
				FM 72	0.000040	0.000040
				FM 73	0.000040	0.000040
8	9	**0.033864**	**0.032501**	FM 81	**0.007820**	**0.007505**
				FM 82	**0.018224**	**0.017491**
				FM 83	**0.007820**	**0.007505**
total		**0.070920**	**0.068399**		**0.070920**	**0.068399**

(d)

Component	Difficulty coefficient	Failure rate		Failure mode	Occurrence rate	
i	δ_i	λ_i^*	λ_i^{**}		λ_{ij}^*	λ_{ij}^{**}
1	5	0.000100	0.000100	FM 11	0.000079	0.000079
				FM 12	0.000015	0.000015
				FM 13	0.000006	0.000006
2	6	0.003807	0.003807	FM 21	0.000276	0.000276
				FM 22	0.003531	0.003531
3	3	0.000600	0.000600	FM 31	0.000351	0.000351
				FM 32	0.000151	0.000151
				FM 33	0.000098	0.000098
4	8	0.030047	**0.028889**	FM 41	0.010168	**0.009776**
				FM 42	0.015532	**0.014933**
				FM 43	0.004347	**0.004180**
5	4	0.000100	0.000100	FM 51	0.000093	0.000093
				FM 52	0.000003	0.000003
				FM 53	0.000004	0.000004
6	7	0.002102	0.002102	FM 61	0.000628	0.000628
				FM 62	0.001474	0.001474
7	**2**	0.000300	0.000300	FM 71	0.000220	0.000220
				FM 72	0.000040	0.000040
				FM 73	0.000040	0.000040
8	9	**0.033864**	**0.032501**	FM 81	**0.007820**	**0.007505**
				FM 82	**0.018224**	**0.017491**
				FM 83	**0.007820**	**0.007505**
total		**0.070920**	**0.068399**		**0.070920**	**0.068399**

From Tables 3(a) to (d), δ_i is adjusted continually and tends to be more reflective of the truth. In four improvement schemes with different δ_i, the numbers in bold mark the items that have been changed. For the series system, $\lambda^* = \sum_{i=1}^{m} \lambda_i^*$ and $\lambda_i^* = \sum_{j=1}^{N_i} \lambda_{ij}^*$ hold [19, 25]. When $\Delta = 0.003630$, $\sum \lambda_i^* = \sum \lambda_{ij}^* = 0.070920$, which is used to verify the correctness of solutions. Additionally, λ_i^* and λ_{ij}^* is hardly sensitive to the changed δ_i while $\Delta = 0.003630$. The reason is that the improvement target of system is relatively small, and the optimal scheme for improvement is almost certain. When $\Delta = 0.006151$, $\sum \lambda_i^{**} = \sum \lambda_{ij}^{**} = 0.068399$, and the correctness of solutions is verified. The sensitivity of λ_i^{**} and λ_{ij}^{**} with respect to the different δ_i would occur while $\Delta = 0.006151$. It is because different components have different improvement difficulties that the different improvement schemes would have different profits.

According to the proposed model, λ_{ij}^* and λ_{ij}^{**} are allocated directly to failure modes, and the allocation results are shown in Table 4. Based on the sensitivity analysis, λ_{ij}^* and λ_{ij}^{**} with respect to different δ_{ij} are listed in Tables 4(a), (b), (c) and (d), respectively.

Table 4. Results of proposed improvement target optimal allocation

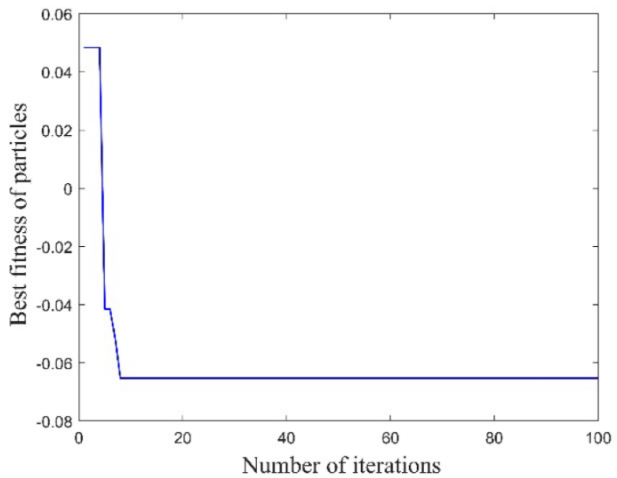
Failure mode	(a)			(b)			(c)			(d)		
	Difficulty coefficient	Occurrence rate		Difficulty coefficient	Occurrence rate		Difficulty coefficient	Occurrence rate		Difficulty coefficient	Occurrence rate	
	δ_{ij}	λ_{ij}^*	λ_{ij}^{**}	δ_{ij}	λ_{ij}^*	λ_{ij}^{**}	δ_{ij}	λ_{ij}^*	λ_{ij}^{**}	δ_{ij}	λ_{ij}^*	λ_{ij}^{**}
FM 11	9	0.000079	0.000079	**5**	0.000079	0.000079	5	0.000079	0.000079	5	0.000079	0.000079
FM 12	9	0.000015	0.000015	**6**	0.000015	0.000015	**7**	0.000015	0.000015	7	0.000015	0.000015
FM 13	9	0.000006	0.000006	**4**	0.000006	0.000006	4	0.000006	0.000006	4	0.000006	0.000006
FM 21	9	0.000276	0.000276	**5**	0.000276	0.000276	5	0.000276	0.000276	5	0.000276	0.000276
FM 22	9	0.003531	0.003531	**6**	0.003531	0.003531	6	0.003531	0.003531	6	0.003531	0.003531
FM 31	9	0.000351	0.000351	**2**	0.000351	0.000351	**3**	0.000351	0.000351	3	0.000351	0.000351
FM 32	9	0.000151	0.000151	**3**	0.000151	0.000151	3	0.000151	0.000151	**4**	0.000151	0.000151
FM 33	9	0.000098	0.000098	**1**	0.000098	0.000098	1	0.000098	0.000098	1	0.000098	0.000098
FM 41	9	0.010168	**0.009742**	**8**	0.010168	**0.009166**	8	0.010168	**0.009166**	8	0.010168	**0.009166**
FM 42	9	**0.011902**	**0.009807**	9	**0.011902**	**0.010383**	9	**0.011902**	**0.010383**	9	**0.011902**	**0.010383**
FM 43	9	0.004347	0.004347	**7**	0.004347	0.004347	7	0.004347	0.004347	7	0.004347	0.004347
FM 51	9	0.000093	0.000093	**4**	0.000093	0.000093	**5**	0.000093	0.000093	5	0.000093	0.000093
FM 52	9	0.000003	0.000003	**4**	0.000003	0.000003	4	0.000003	0.000003	4	0.000003	0.000003
FM 53	9	0.000004	0.000004	**4**	0.000004	0.000004	4	0.000004	0.000004	4	0.000004	0.000004
FM 61	9	0.000628	0.000628	**7**	0.000628	0.000628	7	0.000628	0.000628	7	0.000628	0.000628
FM 62	9	0.001474	0.001474	**8**	0.001474	0.001474	8	0.001474	0.001474	8	0.001474	0.001474
FM 71	9	0.000220	0.000220	**1**	0.000220	0.000220	1	0.000220	0.000220	1	0.000220	0.000220
FM 72	9	0.000040	0.000040	**2**	0.000040	0.000040	2	0.000040	0.000040	2	0.000040	0.000040
FM 73	9	0.000040	0.000040	**3**	0.000040	0.000040	3	0.000040	0.000040	**2**	0.000040	0.000040
FM 81	9	0.008658	0.008658	**8**	0.008658	0.008658	**7**	0.008658	0.008658	7	0.008658	0.008658
FM 82	9	0.020178	0.020178	9	0.020178	0.020178	9	0.020178	0.020178	9	0.020178	0.020178
FM 83	9	0.008658	0.008658	**7**	0.008658	0.008658	7	0.008658	0.008658	7	0.008658	0.008658
Total		**0.070920**	**0.068399**		**0.070920**	**0.068399**		**0.070920**	**0.068399**		**0.070920**	**0.068399**

From Tables 4(a) to (d), δ_{ij} is adjusted continually and tends to be more reasonable and steady, and the numbers in bold mark the items that have been changed. When $\Delta = 0.003630$, $\sum \lambda_i^* = \sum \lambda_{ij}^* = 0.070920$, which is equal to λ^* and justifies the above allocation results. Because $\Delta = 0.003630$ is relatively small, λ_{ij}^* are almost insensitive to the variant of δ_{ij}. When $\Delta = 0.006151$, $\sum \lambda_i^{**} = \sum \lambda_{ij}^{**} = 0.068399$, which equals the value of λ^{**}. Therefore, the results of Table 4 prove to be justified. In addition, λ_{ij}^{**} are only sensitive to major change of δ_{ij}. To some extent, these phenomena reflect the maturity of the proposed optimization model. Moreover, when the same Δ is accomplished, the fewer failure modes need to be improved in Table 4 compared with Table 3. That is to say that the improvement scheme in Table 4 is simpler and more refined.

4.2 Discussion

As a convex function of λ_i^*, the conventional optimal allocation model is solved based on the convex optimization theory in the reference [18]. However, the Eq. (30) as the proposed optimal allocation model is not the convex function under Kuhn-Tucker conditions, so it is difficult to solve directly by the convex optimization theory. In the numerical example, two kinds of optimal allocation models are solved based on the PSO algorithm, so the differences of performance of two models are shown in Tables 3 and 4. The detailed discussions are given as following.

a)



b)

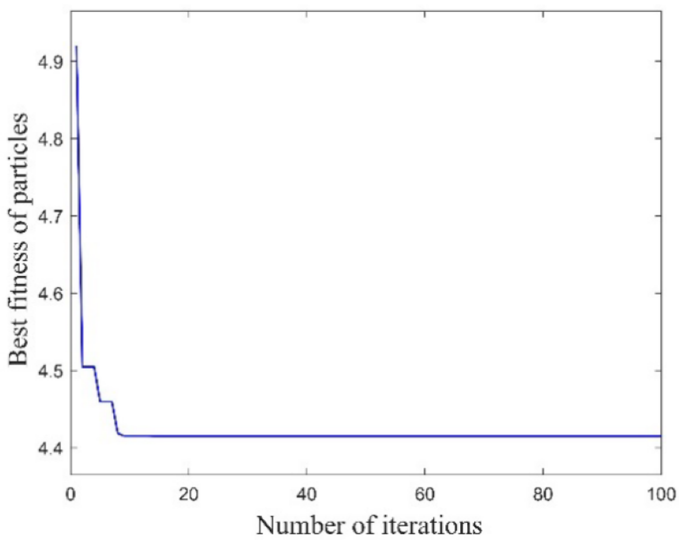


Fig. 6. Optimization processes with δ_i (and δ_{ij})$\in (1,10)$:(a)$\Delta = 0.003630$ in Table 3(d), (b)$\Delta = 0.006151$ in Table 4(d)

Additionally, in Tables 3 and 4, the above optimal allocation results are not very sensitive to the magnified δ_i and δ_{ij}, so the interval of (1, 10) may be more practical in the engineering.

(1) The optimal allocation of system reliability improvement target belongs to multi-objective optimization problem, in which every single objective should be set the corresponding weight. In the conventional method, the system improvement cost is the weighted sum of the component improvement costs, and δ_i playing the role of the weight is set in the interval of (0, 1). However, difficulty coefficients among (0, 1) would cause traps in some examples, such as no convergence of the algorithm. Considering the subjectivity of setting weight, the range of difficulty coefficients is adjusted in this study. As is shown in Tables 3 and 4, both δ_i and δ_{ij} are set in the interval of (1, 10), so that the global optimal solutions of two kinds of models are obtained easily. The optimization processes based on Tables 3(d) and 4(d) with δ_i (and δ_{ij})$\in (1,10)$ are shown in Fig. 6.

(2) Based on Tables 3 and 4, the values of system improvement cost in different models and cases are shown in Table 5.

Table 5. System improvement costs in different models and cases

Model type	$\Delta = 0.003630$				$\Delta = 0.006151$			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
In conventional model	0.916456	0.916456	0.916456	0.916456	1.612710	1.612710	1.600606	1.600606
In proposed model	2.395764	2.395764	2.395764	2.395764	4.523446	4.454549	4.454549	4.454549

As is shown in Table 5, all values of the cost in the proposed model are larger than those in the conventional model. Because $\lambda_i - \lambda_i^*$ is accomplished by decreasing λ_{ij} to λ_{ij}^*, the improvement cost of the component is the sum of the improvement cost of several failure modes. However, $\delta_i ln(\lambda_i/\lambda_i^*)$ not only cannot represent the improvement cost of component i under different improvement schemes, but also is less than the sum of $\delta_{ij} ln(\lambda_{ij}/\lambda_{ij}^*)$. In the proposed formula, these drawbacks are overcome, and the improvement cost of different hierarchies are with definiteness and accumulativeness. In addition, the action of correlation failure leads to a greater complexity for improvement, which causes the higher improvement cost than the independent failure. Therefore, the proposed formula is more reasonable and practical.

Based on Tables 3 and 4, the values of system improvement revenue in different models and cases are shown in Table 6.

Table 6. System improvement revenues in different models and cases

Model type	$\Delta = 0.003630$				$\Delta = 0.006151$			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
In conventional model	0.981661	0.981661	0.981661	0.981661	1.663415	1.663415	1.663415	1.663415
In proposed model	0.018983	0.018983	0.018983	0.018983	0.035947	0.039443	0.039443	0.039443

As is shown in Table 6, the system improvement revenue in the conventional model is larger than those in the proposed model. Due to $s_i = max(s_{i1}, s_{i2}, \ldots, s_{iN_i})$, the conventional model only considers the maximal s_{ij} of component i to calculate the system improvement revenue. On the contrast, the proposed model considers all s_{ij} to calculate the expectation of ΔR_s, and uses all λ_{ij} to calculate the variance of ΔR_s. In addition, the uncertain correlation makes the system risk be random, so it tends to be more difficult to get a high system improvement revenue than the independent failure. Hence, the proposed formula is completer and more reasonable.

(3) On the premise of the correctness of the algorithm and model, the accuracy of the optimal allocation result can be guaranteed. In the proposed model, the unreasonable assumptions are abandoned, such as unique cost of component improvement, constant component severity, independent failure and deterministic risk. Simultaneously, the PSO algorithm allocates λ_{ij}^* to failure modes directly. As is shown in Table 4, Δ is accomplished by decreasing just a few of λ_{ij}. Hence, the allocation result of the propose model is more specific, explicit and refined, and it is more efficient and feasible for the engineering and industry.

5. Conclusions

In this paper, considering multiple correlation failures and risk uncertainty, an optimal allocation of the system reliability improvement target is proposed. A numerical example is given to verify the validity of the proposed method. The conclusions are drawn as following.

(a) By illustrating the concept that the uncertain correlation would cause the disturbance risk, a risk evaluation framework of "actual risk = basic risk + disturbance risk" is proposed under probability measure.

(b) The three kinds of correlation failure are investigated based on the cooperative game theory, and the generalized risk models of the failure mode and system are developed.

(c) Taking the variance of the reduction of the system risk as an additional objective, a multi-objective optimal allocation model is presented, which is solved by using the PSO algorithm.

(d) Through the numerical example and results discussion, the proposed model is proved to be more reasonable, complete and practical, and the solutions of the proposed method are also proved to be more refined and effective.

Acknowledgments

The research was supported by the National Key R&D Program of China (No.2019YFB2006402).

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