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DETERMINATION OF STRESS DISTRIBUTION PATTERNS IN POST-CRITICALDEFORMATION STATES OF THIN-WALLED SKINS SUBJECTED TO OPERATING LOADS

OKREŚLANIE ROZKŁADÓW NAPRĘŻEŃ W STANACH DEFORMACJI ZAKRYTYCZNYCH ELEMENTÓW POKRYĆ CIENKOŚCIENNYCH PODDAWANYCH OBCIĄŻENIOM EKSPLOATACYJNYM*

The paper presents results of research on thin-walled structures constituting models of skin components of aircraft load-bearing structures subject to shearing. The type of load and deformation correspond to the typical state of such a structure under operating conditions. Results of experiments carried out on models and nonlinear numerical analyses based on the finite element method for a number of load-bearing structure variants. Methods suitable for verification of results of numerical calculations are proposed and an effective set of numerical procedures is selected allowing to obtain reliable results of calculations.

Keywords: *buckling, thin-walled structures, aircraft load-bearing structures, shearing, finite element method, nonlinear numerical analyses, operating stability.*

Praca prezentuje wyniki badań ustrojów cienkościennych, stanowiących modele fragmentów pokryć lotniczych struktur nośnych, poddawanych ścinaniu. Rodzaj obciążenia oraz deformacji odpowiada typowemu stanowi struktury w warunkach eksploatacji. Przedstawiono wyniki badań eksperymentalnych i nieliniowych analiz numerycznych w ujęciu metody elementów skończonych szeregu wariantów ustrojów nośnych. Opracowano metody weryfikacji wyników obliczeń numerycznych oraz dobrano efektywny zestaw procedur numerycznych pozwalających na uzyskanie wiarygodnych wyników obliczeń.

Słowa kluczowe*:* *wyboczenie, ustroje cienkościenne, lotnicze struktury nośne, ścinanie, metoda elementów skończonych, nieliniowe analizy numeryczne, trwałość eksploatacyjna.*

1. Introduction

The loss of stability in thin-walled structures subjected to shearing is a phenomenon commonly encountered in the course of operation of semi-monocoque aircraft structures. Aircraft skin components lose their stability in the course of operation, and therefore in the permissible load conditions. The phenomenon is considered permissible in case of metal structures provided the deformation occurs in the elastic regime and occurs locally, i.e. within the skin areas limited with frame components [16].

The aircraft structure design processes must therefore include, as their indispensable steps, analyses allowing to determine the stress distribution patterns in advanced post-buckling deformation states that, in the case of skins being components of semi-monocoque aircraft structures, have cyclical nature in view of variability of loads. This results in occurrence of fatigue-related effects in the skin material, therefore the knowledge of stress fields is essential not only for the purpose of identification of stress concentration zones, but constitutes also a base on which operating stability of the analysed structures will be evaluated [14, 15].

Additional problems are caused by cut-outs of any type, such as service sight holes and inspection openings, that result in local reduction of rigidity of the structure. They are therefore a cause of the change of the stability loss characteristics with respect to areas free from such singularities and result in local occurrence of strong stress concentrations [1, 4, 5].

Basic numerical tools widely used throughout design processes are various computer programs utilising the finite element method (FEM) [3, 6, 18]. Whereas in the case of linear analyses and at the presently observed state of advancement of commercial software, most of the obtained results may be admitted to be reliable, nonlinear procedures used for determination of advanced post-critical deformation states are still the source of many problems following from limited reliability of numerical procedures determining equilibrium states of structures [7, 19] corresponding to consecutive points of the equilibrium path [2, 12, 13]. As a consequence, despite apparently correct representation of structure rigidity, results biased with significant errors are frequently obtained [11].

From the point of view of description of a nonlinear problem, the fundamental relationship determining the state of a structure versus the load is the so-called equilibrium path of the system in question, interpreted in general as a hypersurface in the hyperspace of states [6, 8]. The relationship represents fulfilment of the equation of residual forces in a matrix form [3, 12]

$$\mathbf{r}(\mathbf{u}, \Lambda) = \mathbf{0}, \tag{1}$$

where **u** is the state vector containing displacement components of nodes of the structure corresponding to its current geometrical configuration, Λ is a matrix composed of control parameters corresponding to the current load level, and **r** is the residual vector composed of uncompensated force components related to the current system

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(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** deformation state. The set of control parameters can be represented by a single parameter being a function of the load. Equation (1) takes then the form

$$\mathbf{r}(\mathbf{u}, \lambda) = \mathbf{0}, \tag{2}$$

called the single-parameter equation of residual forces.

The prediction-correction methods of determining consecutive points of the equilibrium path used in modern software routines contain also a correction phase based on the requirement that the system satisfies an additional equation called the increment control equation or the equation of constraints [7]:

$$c(\Delta\mathbf{u}_n, \Delta\lambda_n) = 0, \tag{3}$$

where the increments

$$\Delta\mathbf{u}_n = \mathbf{u}_{n+1} - \mathbf{u} \quad \text{and} \quad \Delta\lambda_n = \lambda_{n+1} - \lambda_n \tag{4}$$

correspond to the transition from n-th state to $(n+1)$-th state.

For the purpose of comparison, in view of the obvious difficulties involved in graphical representation of equilibrium paths for systems with more than 2 degrees of freedom, in practice the so-called representative equilibrium paths are used that represent a functional relationship between a selected parameter characterising deformation of the system and a single control parameter related to the applied load.

To confirm reliability of results obtained from FEM-based nonlinear numerical analyses, it is necessary to find a satisfactory convergence between representative equilibrium paths, the actual one and this obtained numerically, and further, confirm correctness of the deformation pattern resulting from calculations [10]. It is therefore essential to carry out the process of experimental verification. Selection of a relatively inexpensive experiment with the use of model materials seems to be particularly recommendable in this case.

On the grounds of the solution uniqueness rule, according to which a specific deformation pattern may correspond to one and only one stress distribution pattern, reliability can be then attributed also to the reduced stress distributions in the deformed skin [9, 10].

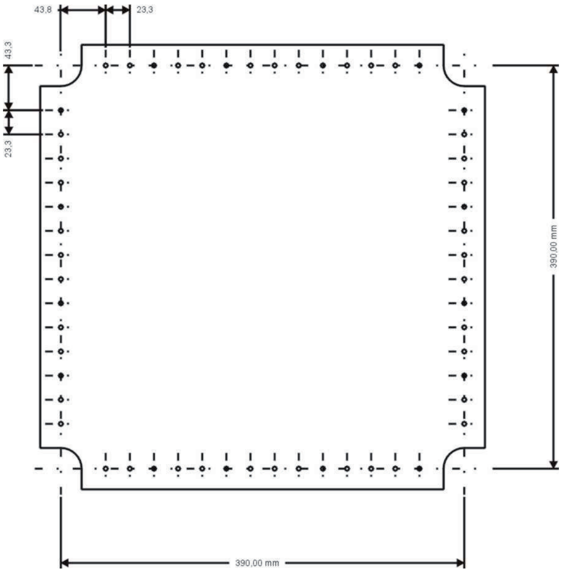


Fig. 1. Geometry of the examined structure

2. Subject and scope of research

The presented study was aimed at development of an effective methodology applicable to determination of stress distribution patterns in flat components of skins subjected to post-critical deformations under operating conditions, with the stress put on selection of appropriate numerical methods and parameters controlling the course of nonlinear analyses carried out with the use of the finite element method.

The subject of the study were thin-walled structures with shape and dimensions shown in Fig. 1 subjected to shearing and representing model counterparts of typical skin components used in semi-monocoque structures. The analysis covered also structures provided with circular holes with various diameters imitating service openings.

2. Experimental research

According to the assumptions, the planned research cycle included several stages realisation of which required construction of several test set-ups.

Fig. 2. The set-up for static experiments with 3D ATOS scanner

The basic set-up, used for experiments of static character, comprised a frame with a large margin of rigidity on which both structure mounting system and loading system were supported (Fig. 2).

The examined structure was mounted in a special frame made of rigid steel beams joined pivotally in corners. The load in the form of a force was applied gravitationally, by means of a line put over a system of pulleys (Fig. 3). To be able to perform the above-mentioned comparison between deformations obtained by means of numerical methods and the actual ones, it turned out to be necessary to make the deformed structure geometry measurements as accurate as possible. For the purpose of the present study, a set of ATOS scanners manufactured by German company GOM Optical Measuring Techniques was used. The principle of operation of the scanners is based on the projection moiré method, and geometry measurements were carried out after each increment of the applied load.

The experiment was carried out with the use of five model variants: the basic structure without any cut-outs, two structures with centrally located circular holes with diameters of 35 mm and 70 mm, and

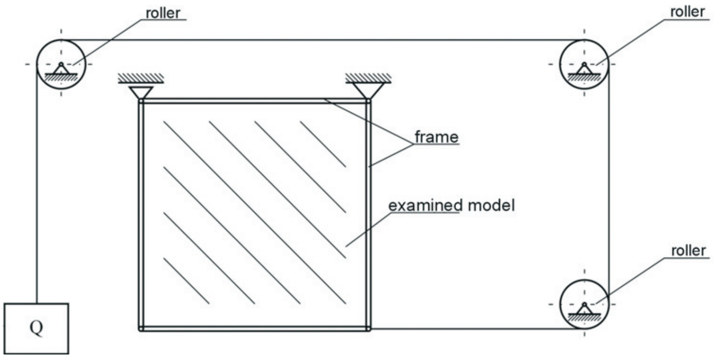


Fig. 3. Schematic diagram of fastening and load application

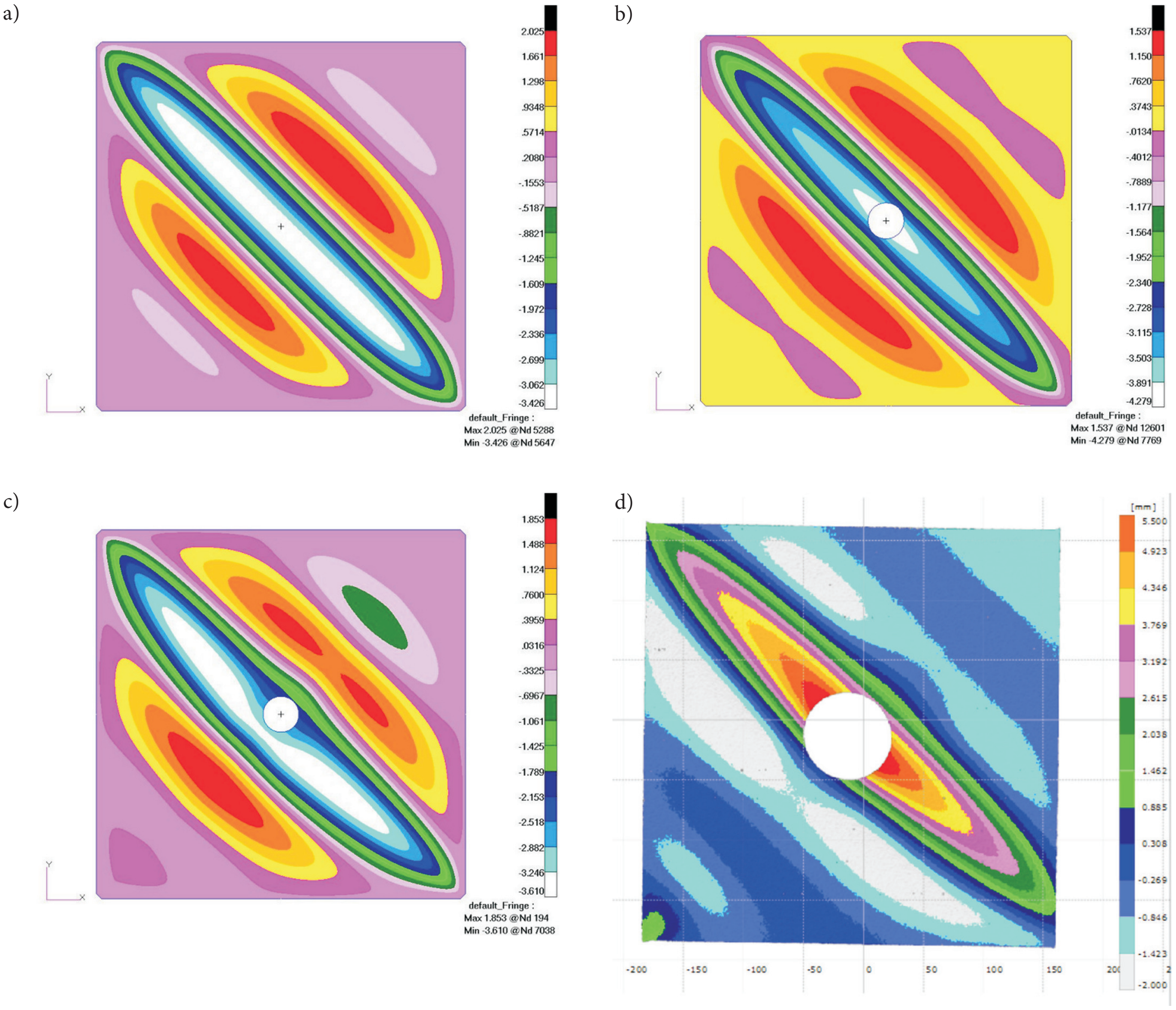


Fig. 4. Displacement distributions obtained as a result of experiment with the use of polycarbonate boards: (a) the model without any cut-out; (b) a model with 35-mm hole without reinforcement; (c) a model with 35-mm hole and reinforcing ring; (d) a model with 70-mm hole

two structures with holes of the same diameters but provided with reinforcing rings 5 mm wide and 4 mm thick. In all the cases, the examined structures were loaded with force the maximum value of which reached 900 N. The models used in the first phase of research were made of polycarbonate marketed under trade name of Macrolon, which is an isotropic material characterised with the following parameters: the Young's modulus E = 2150 MPa, and the Poisson ratio ν = 0.4.

The loss of stability occurred in all the cases at relatively low load values which follows from the nature of the stress distribution pattern constituting the field of tensions. The displacement distributions obtained in the course of the experiment are presented in Fig. 4 in the form of colour-filled contour lines generated with the use of GOM Inspect Software linked with ATOS 3D scanner.

In all the cases, advanced post-critical deformations had a similar form, with a fold running along the diagonal of the model. It should be e)

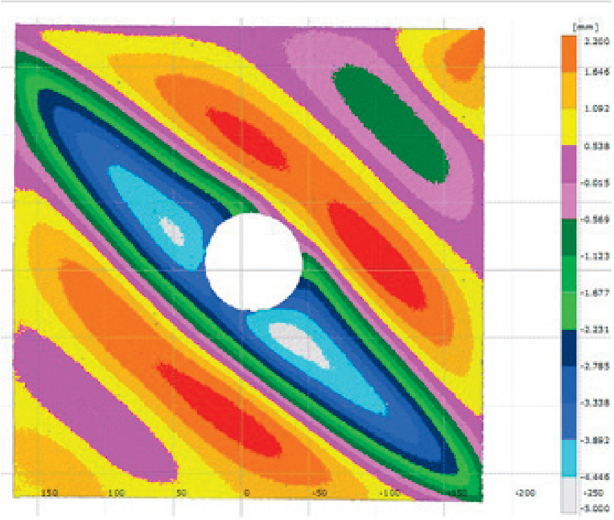


Fig. 4. [continued] (e) a model with 70-mm hole and reinforcing ring

however emphasised that the examined objects are characterised by high susceptibility to any changes occurring in boundary conditions which is typical for nonlinear systems. One of such changes consists in introduction of additional stiffening elements around the holes. In these cases, a change in the deformation pattern was observed in the form of shifting the folds towards the corner which was a result of local stiffening of the structure. For all the cases, representative equilibrium paths have been obtained by means of establishing the relationship between the maximum displacement along direction perpendicular to the undeformed model plane and the load force value (Figs. 11–15).

In the subsequent experiment, a plate made of an aircraft aluminium alloy was used. As a part of the experimental set-up, a load-applying system was used in the form of numerically controlled hydraulic servo by Zwick (Fig. 5). The model was mounted by means of the same method as this used in the case of polycarbonate boards. Utilisation of the modified system was aimed at provision of the possibility to carry out research on both static and fatigue-related phenomena which would allow to determine relationships between stress distributions occurring under the static load conditions and operating stability of the examined structures.

Fig. 5. A set-up for static and fatigue tests with a model made of aluminium alloy plate

In the course of the experiment, geometry of the deformed model was registered with the use of ATOS scanner (Fig. 6).

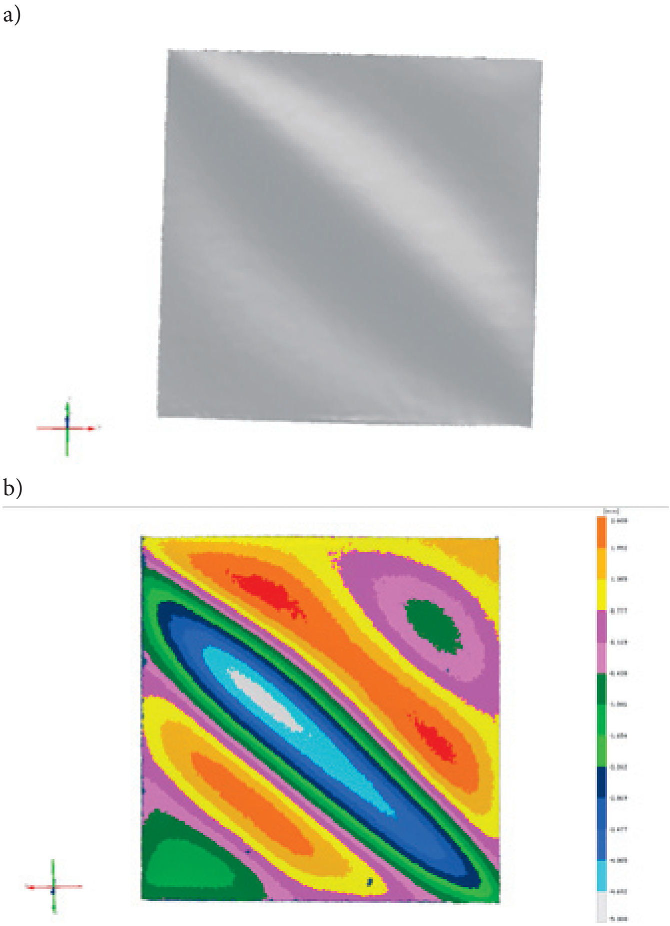


Fig. 6. Deformation of the plate made of aluminium alloy: (a) an overview, (b) distribution in the form of contour lines

3. Nonlinear numerical analysis

Numerical calculations were carried out with the use of MSC Patran / MSC Marc software. The examined structures were modelled by means of about 5000 thin-walled (thin shell) elements. To model the mounting system, about 2000 thick-walled (thick shell) elements were used. Joints of continuous character have been assumed between the mounting system and the model of the examined structure. This simplification followed from striving to obtain a task of relatively small size and from the fact that using MPC nodes selected typically as a representation of discrete joints results in occurrence of various errors in nonlinear numerical procedures leading to distortion of final results or even making impossible to find any solution at all. After a series of numerical tests it has been found that the best results, both qualitatively and quantitatively, would be obtained by using the prognostic secant method in combination with the Crisfield's hyperspherical correction strategy [8]. Moreover, appropriate values of the parameters controlling the nonlinear process have been selected. Fig. 7 presents selected numerical models of the examined structures with the mesh of divisions into finite elements shown.

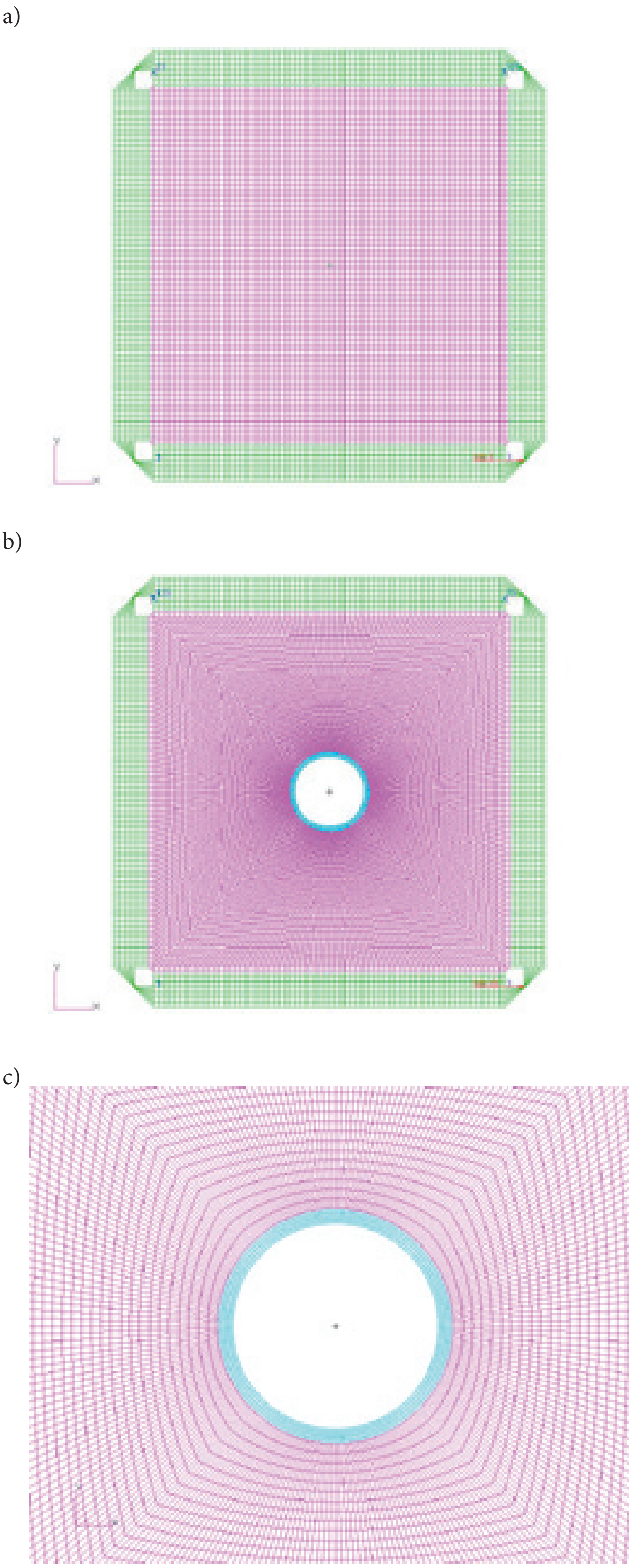


Fig. 7. Selected numerical models of the analysed structures: (a) the model without any cut-out; (b) the model with 70-mm hole and reinforcing ring; (c) the zone of 35mm hole with reinforcing ring

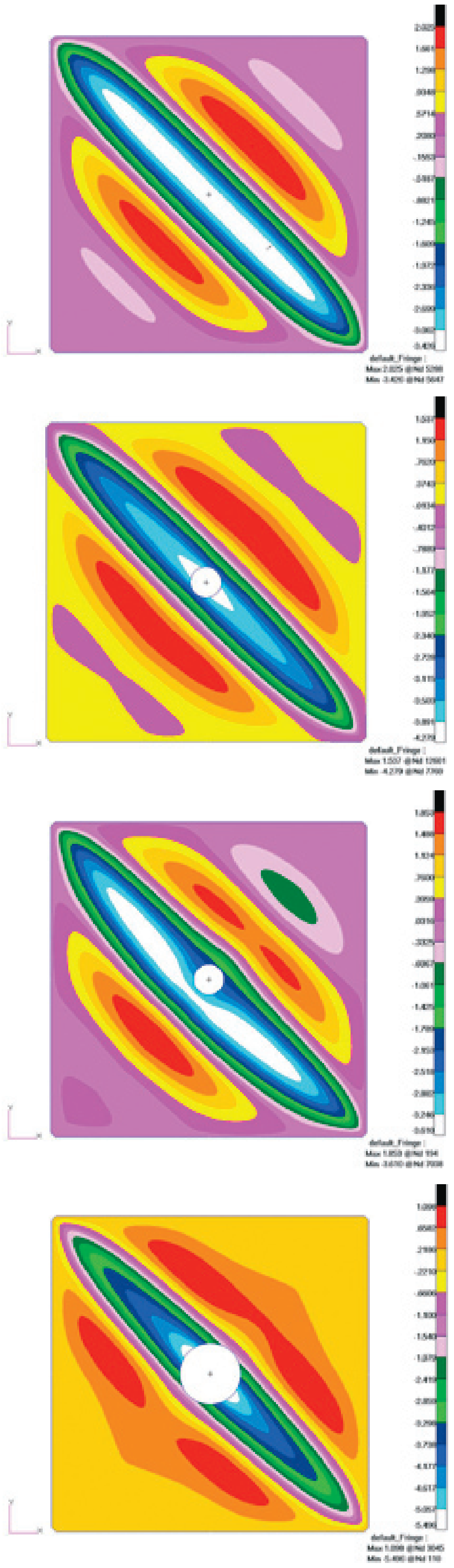


Fig. 8. Displacement distribution patterns along direction perpendicular to the undeformed model plane – polycarbonate skins

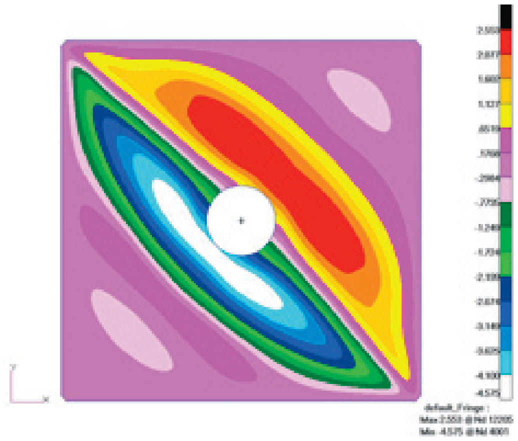
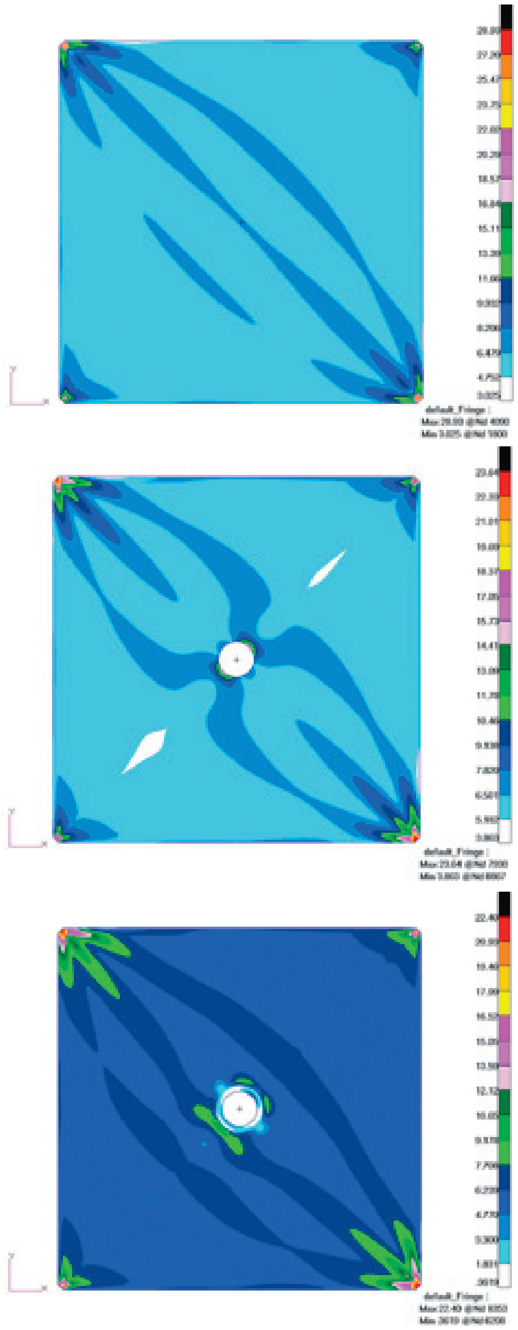


Fig. 8. [continued] Displacement distribution patterns along direction perpendicular to the undeformed model plane — polycarbonate skins



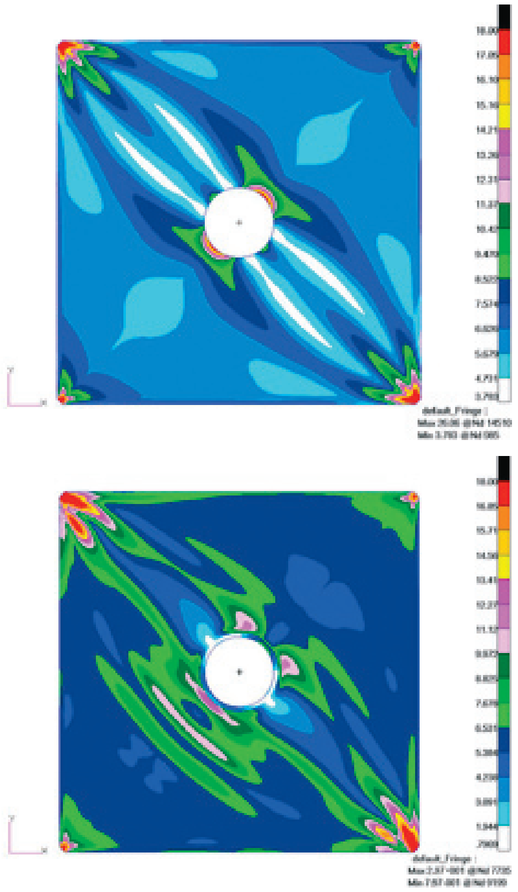


Fig. 9. Reduced stress distributions according to Huber-von Mises-Hencky hypothesis – polycarbonate skins

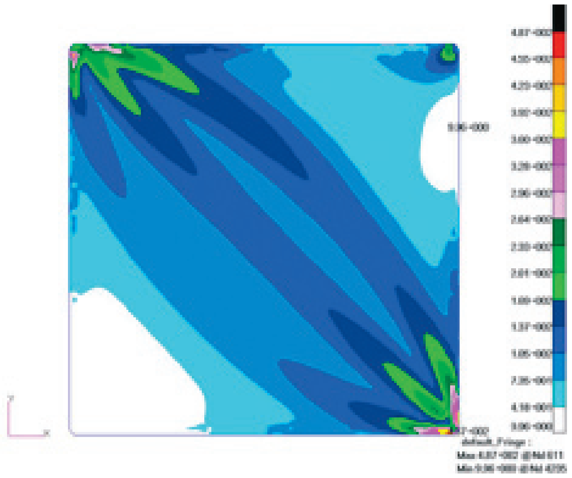


Fig. 10. Reduced stress distribution according to Huber-von Mises-Hencky hypothesis – aluminium alloy plate

Fig. 8 shows deformation distributions obtained by means of numerical methods, while Figs. 9 and 10 depict reduced stress distributions according to Huber-von Mises-Hencky hypothesis in the analysed models.

Figures 11–15 show comparisons between representative equilibrium paths obtained from experiments on one hand and nonlinear numerical analyses on the other. In all the cases, as a representative parameter, the component of displacement of point A (see Fig. 3) corresponding to X axis of the reference system was adopted.

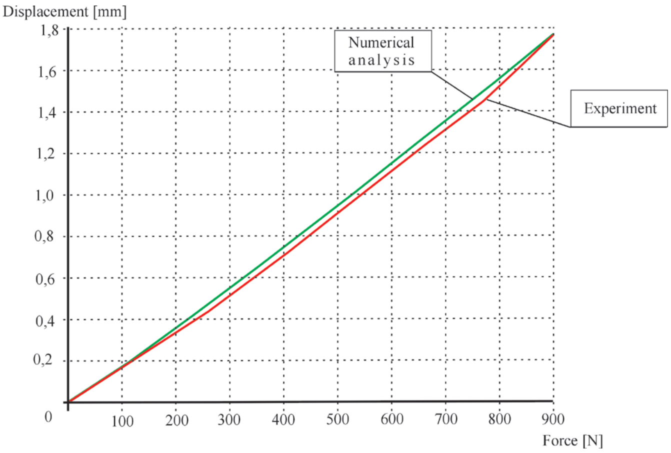


Fig. 11. A comparison between representative equilibrium paths – the model without hole

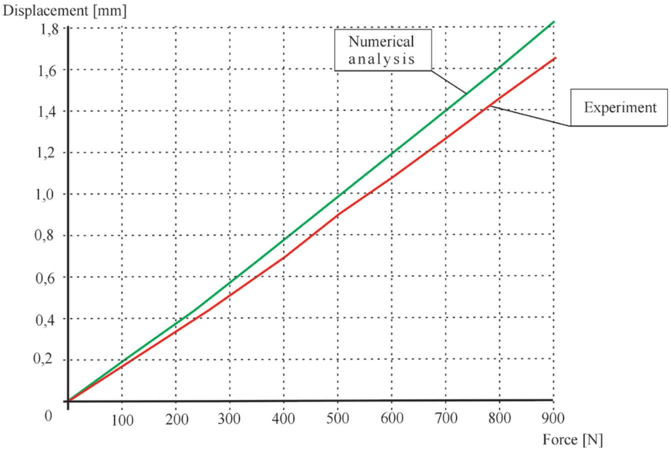


Fig. 12. A comparison between representative equilibrium paths – the model with a 35-mm hole

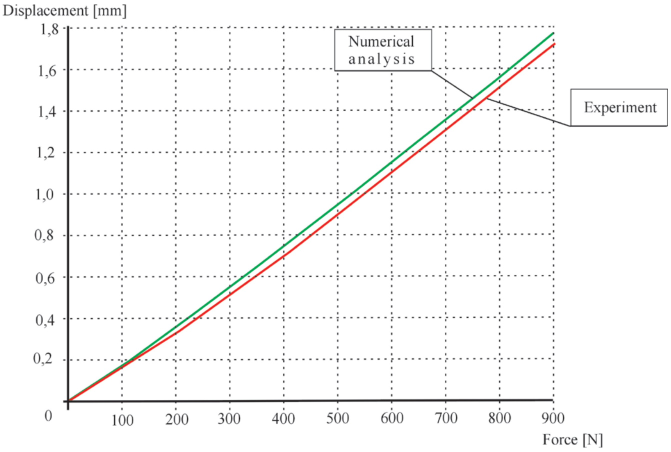


Fig. 13. A comparison between representative equilibrium paths – the model with a reinforced 35-mm hole

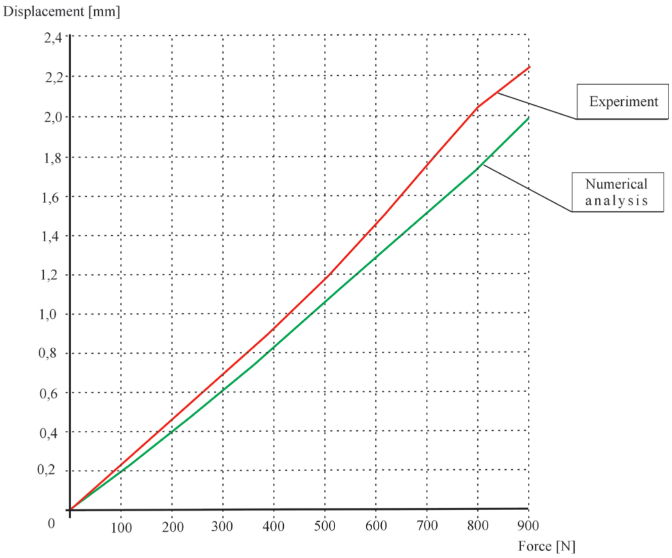


Fig. 14. A comparison between representative equilibrium paths – the model with a 70-mm hole

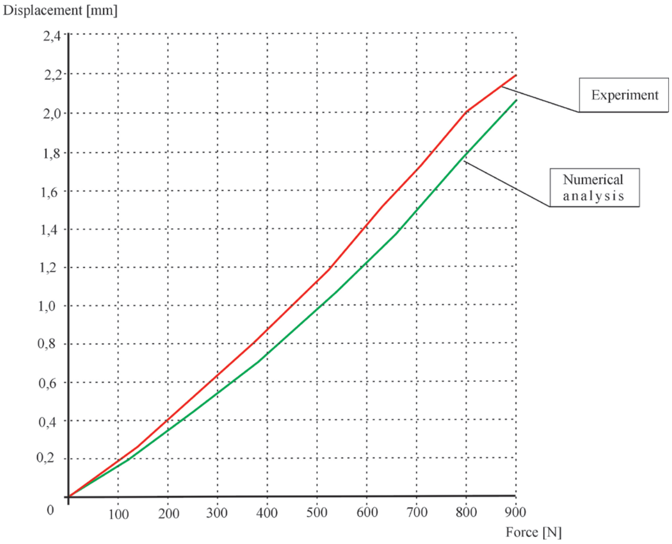


Fig. 15. A comparison between representative equilibrium paths – the model with a reinforced 70-mm hole

4. Discussion on the results

According to the adopted criteria, the assessment of reliability of results obtained from nonlinear numerical analysis was based on similarity of deformation distribution patterns (Figs. 4 and 8). Taking into account the practically identical overall deformation picture and very similar displacement values in corresponding areas of the two plate types, it may be concluded that both the adopted prognostic method and the correction strategy constitute a combination suitable for analysis of structures similar to these discussed above. Reliability of the obtained results is confirmed by satisfactory convergence of representative equilibrium paths obtained from experiments and numerical models, for each of the analysed structures (Figs. 11–15).

Based on the above-described mutual correspondence, reliability has been also admitted with respect to the reduced stress distributions presented in Figs. 9 and 10. In all these cases, stress concentration zones were located in corners of the examined structures and these are the locations where occurrence of a fatigue damage can be expected.

In all the cases discussed above, increased values of the reduced stress values occur along boundaries of folds which follows from flexural states arising in the examined structures. Moreover, stress concentration zones develop in the areas adjacent to holes, whereas in the case of cut-outs with reinforcements, reduced stress gradients and maximum values are much less than in the case of non-reinforced holes. Increase of the hole diameter results in a natural way in an increase of stresses occurring in the plate and necessity to use reinforcements with respectively increased dimensions.

The above observations allow to conclude that the search for structural design solutions aimed at improvement of operating stability of thin-walled structure skins should be oriented towards determination and elaboration of alternative solutions allowing to change the nature of stress distribution patterns. An example of solutions of that type used in the aircraft construction practice are skin stiffening features in the form of the so-called ripples, i.e. parallel mouldings within the area of skin segments or, much more rarely, integral stiffening elements obtained by means of machining. Such solutions, although used in the practice, are rather scarcely discussed in universally available scientific publications in view of high costs of the related research work that is in general carried out only by aircraft manufacturing corporations for their own purposes.

5. Directions of further research

The research results presented above together with observations based on them should be perceived in the context of a wider research program aimed at development of effective calculation methods that would allow to evaluate stability of thin-walled aircraft structures subjected to post-critical deformations under permissible operating load conditions. Selection of appropriate methods to be used for development of numerical models and the most effective combinations of prognostic methods and correction strategies in nonlinear analyses allows to work out fail-safe methods for determining stress distributions in deformed skins. The obtained stress distributions may be adopted as a base for further analyses of fatigue-oriented nature taking into account the use of appropriate software and performance of a relevant model experiment.

The proposed research program represents therefore a combination of static and fatigue experiments carried out with the use of up-to-date research apparatus, e.g. a hydraulic cylinder for fatigue tests, as well as numerical analyses of various types, with the use of a up-to-date software based on the finite elements method. The fundamental objective of such research effort is to develop a computational method for determining the operating stability applicable to various variants of thin-walled structure skins subjected to large deformations under permissible load conditions.

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