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NUMERICAL ANALYSIS OF COMPRESSED PLATES WITH A CUT-OUT OPERATING IN THE GEOMETRICALLY NONLINEAR RANGE

NUMERYCZNE BADANIA PRACY ŚCISKANYCH ELEMENTÓW PŁYTOWYCH Z WYCIĘCIEM W ZAKRESIE GEOMETRYCZNIE NIELINIOWYM*

This paper presents the results of a numerical analysis conducted to investigate uniformly compressed rectangular plates with different cut-out sizes. Made of high strength steel, the plates are articulatedly supported on their shorter edges. The FEM analysis examines the nonlinear stability of these structures in the post-buckling state, where the mode of buckling is forced to ensure their stable behaviour. The numerical computations are performed within the geometrically nonlinear range until the yield point is reached. The investigation involves determining the effect of cut-out sizes on elastic properties of the plates with respect to service loads. The numerical analysis is conducted using the ABAQUS software.

***Keywords**: thin-walled structures, finite element method, numerical analysis, thin-walled elastic elements, plate stability.*

Przedmiotem badań są prostokątne płyty z wycięciem o zmiennych parametrach geometrycznych poddane równomiernemu ściskaniu. Płyty podparte przegubowo na krótszych krawędziach wykonano ze stali o wysokich właściwościach wytrzymałościowych. Badania dotyczyły numerycznej analizy MES nieliniowej stateczności konstrukcji znajdujących się w stanie pokrytycznym z wymuszoną postacią wyboczenia zapewniającą stateczny charakter pracy konstrukcji. Obliczenia prowadzono w zakresie geometrycznie nieliniowym do uzyskania poziomu granicy plastyczności materiału. Badano wpływ parametrów geometrycznych wycięcia na charakterystykę sprężystą płyty w zakresie obciążeń eksploatacyjnych. Zastosowanym narzędziem numerycznym był program ABAQUS.

***Słowa kluczowe**: konstrukcje cienkościenne, metoda elementów skończonych, analiza numeryczna, cienkościenne elementy sprężyste, stateczność konstrukcji płytowych.*

1. Introduction

Thin-walled load-carrying structures are characterized by high strength and rigidity as well as low weight. Given the above properties, these structures are widely applied not only in the aerospace and automotive sectors, but also in designs where low structure weight is crucial. A disadvantage of thin-walled structures is that they are prone to loss of stability under compressive or shearing loads [2, 3, 9, 13, 21–25, 28, 29]. Nonetheless, thin-walled structural elements can operate even after their stability loss provided that they do it in the elastic range [11, 14, 21, 27]. These structures are particularly sensitive to geometric inaccuracies, which is critical regarding their use. For this reason, they must be precisely manufactured to prevent premature loss of stability. Uniform thin-walled plates are relatively cheap to produce, yet they can carry relatively low loads due to their low flexural rigidity. When subjected to compression, these plates lose stability even under small loads. With common methods for improving load capacity such as the use of stiffeners or ribs, the form of a structure is not only considerably modified, but its weight can increase, too. However, there is a way in which load capacity of thin-walled structures can be greatly improved while the structures themselves can be used as both carrying structures and elastic elements. This can be done by forcing these structures to operate in a higher buckling mode (flexural and torsional). To do so, it is necessary to make a cut-out in the plate and slightly displace the vertical zone in the opposite direction. This displacement will be realized in a special frame - so that the jump on basic mode was impossible. In this way, the plate acquires the desired characteristics, i.e. stable operation in the post-buckling range.

Issues of the stability and post-buckling behaviors of plate structure with all sorts of holes was described in [19], where you can find a very comprehensive review of the literature on this issue. At that work as the author of the earliest elaboration, in numerical terms, is given Pennington Vanna [16], who as first considered the problem of the stability of elastic uniaxially compression plate with a cut-out. The article presents the results of FEM calculations and compared to the results of experimental studies. The earliest post-buckling behavior analysis of plate structures with cut-out are reported in [18]. A major contribution brings the works [15, 17, 20, 30], where are considered the postcritical states and limited carrying capacity of this type of structures. Not encountered of research, where was investigated plate structures with cut-out and in which attempted to force of the higher flexural and torsional mode deformation of the structure.

In the researched load-carrying structure, one can clearly distinguish a vertical zone where this thin-walled element undergoes compression and bending as well as a horizontal zone where it is mainly subjected to torsion. Plate characteristics vary depending on the area of these zones. This observation is critical with respect to plate opera-

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(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** tion as it indicates that it is possible to produce elements with identical dimensions yet totally different rigidity.

The design of machines and devices sometimes requires the use of elements that will protect them from damage. Such elements should have low weight and specific operating characteristics. Apart from that, machine design often requires the use of elastic elements that need to be built in a rectangular space. In either cases, plates with a cut-out can be applied.

The investigation involved examining a thin-walled plate with a cut-out to be used as an elastic or carrying element. We investigated the effect of cut-out sizes on the characteristics of plate operation with respect to service loads. The numerical analysis was performed by the finite element method.

2. Research object and scope

The analysis was performed on rectangular plates with different cut-out sizes. The plates were made of spring steel 50HS and had the following material properties: a Young's modulus, E, of 210000MPa, a Poisson's ratio, v, of 0.3, a yield point, $R_{e,}$ of 1180 MPa and a strength limit, R_m, of 1320 MPa. In all examined cases, the plates had the same overall dimensions: a height, H, of 250 mm, a width, B, of 150 mm and a thickness, g, of 1 mm (Fig. 1). The investigated plates had a centrally located, symmetric cut-out, the geometric variables of which, i.e. height a and width b, exerted a significant effect on the structure's behaviour under load. The investigated range of the cut-out geometric variables was respectively: $a = 80 \div 200$ mm and $b = 10 \div 50$ mm, which meant examining 35 cases of cut-out geometry.

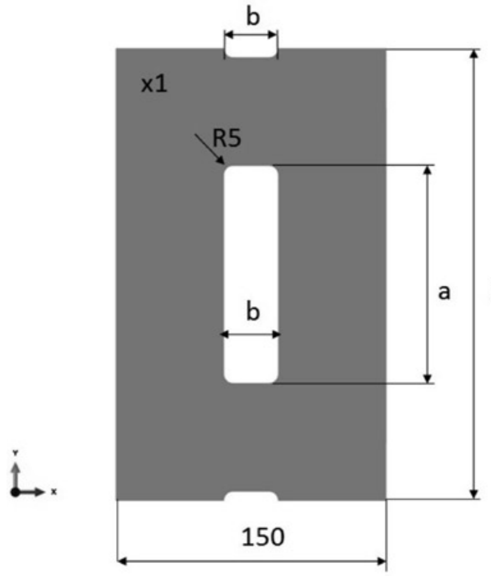

Fig. 1. Geometric dimensions of a plate with a cut-out

The investigation involved performing a numerical analysis of non-linear stability of a uniformly compressed plate, where a higher mode of buckling was forced to ensure stable operation of this structure in the post-buckling state. In order to examine the desired buckling mode, the plate had a central cut-out, the geometric dimensions of which had a direct effect on the plate's stability and operation in the post-buckling state. As a result, they affected the characteristics of the plate's post-buckling equilibrium path in the elastic state. This way of shaping elastic properties of plates is particularly important if such structures are to be used for various designs. Owing to considerable displacements that occurred when the plate was under load, the numerical analysis also involved solving the problem of geometric nonlinearity using the Newton-Raphson method [1, 4, 5]. The numerical analysis was performed using the FE-based commercial software ABAQUS® [1].

3. Numerical analysis

The discretization of the tested plate was made using the eight-node reduced integration shell elements (S8R), each element having six degrees of freedom at every node. In these thin-walled shell elements, the strains corresponding to the membrane state are determined based on linear displacements, while the flexural strains are defined based on the angular displacements [1, 8]. In the analysis, we used elements with the second order shape function**.** Fig. 2 shows the general view of the numerical model of the structure.

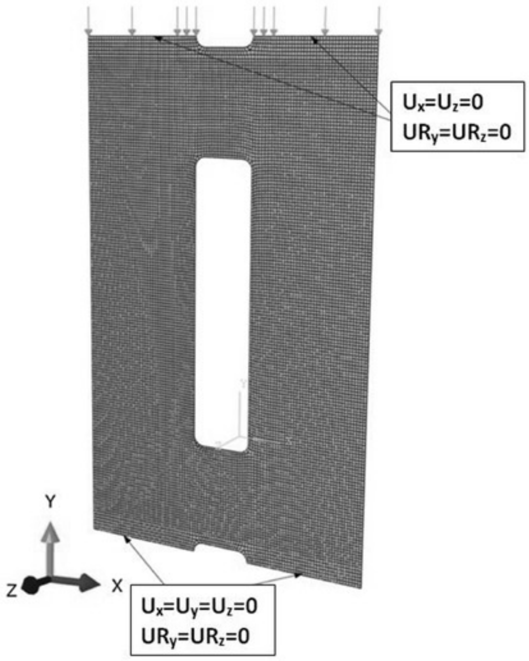

Fig. 2. Discrete model of a plate with a central cut-out

The boundary conditions of the numerical model which plotted the articulated support of the plate were defined by blocking the kinematic degrees of freedom of the nodes located on the upper and lower edges of the plate. The FEM model was loaded by applying uniform load to the upper edge of the plate.

The numerical analysis assumed that the operating range of the elastic element is set below the yield point and it does not alter the original rigidity of this element. Thus, the computations were continued until reaching the yield point, i.e., R_e=1180 MPa. The operational range of the structure was defined by the linear and elastic material model.

The numerical computations were run in two stages. The first stage involved examining the buckling state of the structure. The solving of this problem consisted in determining the buckling load and the corresponding mode of stability loss. For every case, we determined 3 lowest buckling modes, which allowed us to determine the flexural and torsional buckling mode that would ensure stable operation of the structure after buckling. The second stage of the computations involved solving the problem of nonlinear stability. The computations were made using models with initiated geometric imperfection which corresponded to the flexural and torsional buckling mode [10, 12]. In the computations, the amplitude of initial imperfections was set to 0.1 of the plate thickness.

4. Discussion of the numerical results

The numerical analysis enabled the determination of post-buckling equilibrium paths of compressed rectangular plates depending on the cut-out geometric variables: height *a* and width *b*. The numerical calculations were conducted for the buckling mode that ensured stable operation of the structure in the post-buckling range (Fig. 3).

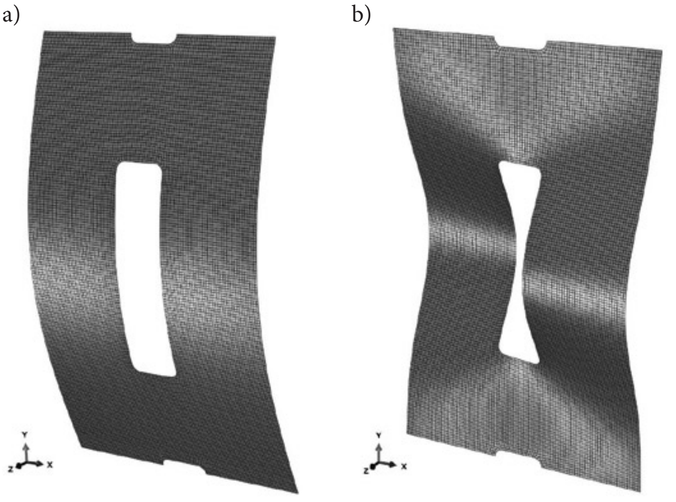

Fig. 3. Modes of stability loss in a plate with a cut-out: a) initial mode, b) higher mode

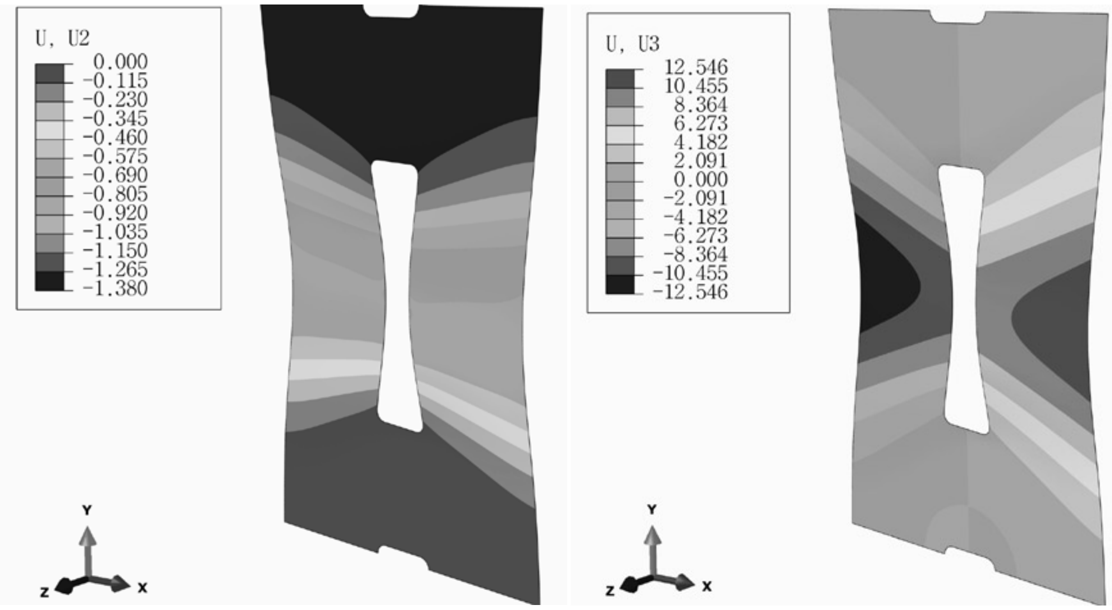

Fig. 4. Plate deformation in the post-buckling state: a) vertical deflection, b) lateral deflections

The buckling mode applied in the non-linear stability analysis constituted a higher eigenvalue and it corresponded to the flexural and torsional mode, one that forced non-symmetric deflections of the plate opposite to the cut-out. In effect, the load-carrying capacity of the structure was greatly improved compared to the same-sized uniform plate wherein the loss of stability corresponded to the lowest eigenvalue (flexural buckling). An example of the distribution of displacements in the plate with the cut-out is shown in Fig. 4.

The numerical results allowed us to perform both qualitative and quantitative analysis of the structure's behaviour in the post-buckling range. In all investigated cases, the post-buckling modes were made to be higher forms of the implemented buckling mode until the yield point was reached when the structure began to deform plastically, only to fail in the end. For this range, we developed post-buckling equilibrium paths, *P-U2*, in order to illustrate the relationships between load and vertical deflection of the plate edge. Thereby determined characteristics allowed us to evaluate the structure's operation depending on the geometric variables *a* and *b*. Fig. 5 shows some of the structure's operating characteristics versus cut-out heights in the tested plates (variable *a*). The curves reveal that – with the plate overall dimensions maintained unchanged – one can obtain structures with totally different operating characteristics, depending on the cut-out sizes.

The quantitative analysis of the results revealed a high discrepancy regarding the load capacity at different cut-out heights, ranging from 1003.1 N for a 200 mm high cut-out (50×200 plate) to 5000 N for a 80 mm high cut-out (10×80 plate). The cut-out width had greatest effect on 100 mm high plates where the highest difference in load capacity amounted to 1637.4 N. The results confirm that the cut-out height *a* has a significant effect on the post-buckling characteristics of the tested plates. This is important from a practical point of view as it means that - when it comes to structures with elastic elements – we can produce thin-walled structures with the required operating characteristics. Detailed results concerning the allowable operating ranges of the plate in elastic state, depending on the cut-out dimensions, are listed in Table 1. A nearly fivefold difference in load capacity upon the reaching of the yield point proves that the characteristics of elastic elements designed thereby can, to a great extent, be shaped as desired.

The operational ranges of the tested structures are illustrated in Fig. 6. The results demonstrate that the cut-out variables are relatively easy to select so as to produce a plate with the required rigidity in a wide range of load.

Fig. 7 shows the plot of reduced stress determined using the Huber-Mises-Hencky (H-M-H) hypothesis for the load that causes reaching the stress level which corresponds to the yield point and fro individual stress state components (plate with a 30x120mm cut-out). The distributions of reduced stress indicate that the corner of the cut-out is the crucial region of the structure, since it determines this structure's

Table 1. Maximum load capacity the plate [N] depending on the cut-out variables a and b – loads causing plastic deformation of the plate

Cut-out width *b* [mm]	**Cut-out height *a* [mm]**						
	80	**100**	**120**	**140**	**160**	**180**	**200**
10	5000.0	4600.0	3529.3	2854.1	2318.1	1909.6	1558.9
20	5000.0	4017.9	3166.9	2560.6	2106.5	1721.4	1405.2
30	4707.6	3613.4	2861.6	2313.7	1905.4	1558.4	1250.0
40	4252.4	3267.7	2605.0	2107.5	1720.0	1400.0	1113.5
50	3851.5	2962.6	2316.0	1907.7	1558.1	1261.0	1003.1

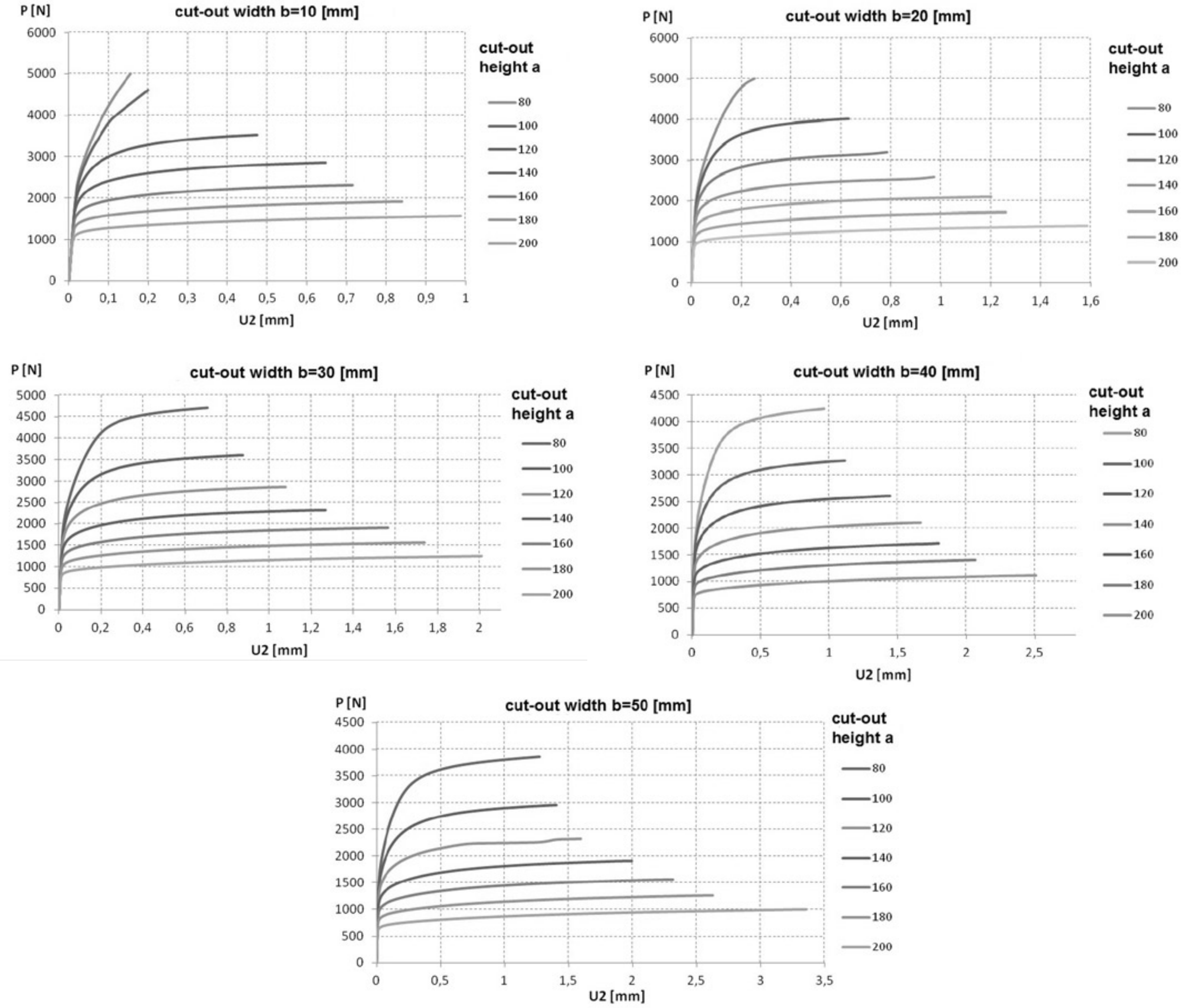

Fig. 5. Post-buckling equilibrium paths P-U2 versus height of the cut-out

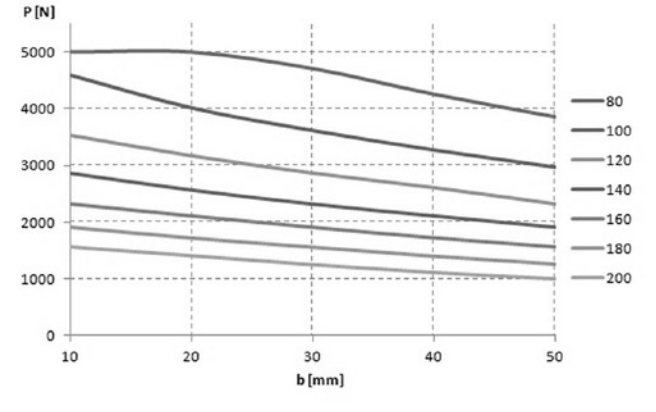

Fig. 6. Maximum load capacity of the plate [N] versus the cut-out variables a and b

operation in the elastic range. It is in this region that the material begins to deform plastically; therefore – as assumed – the deforming load is defined as the limit service load.

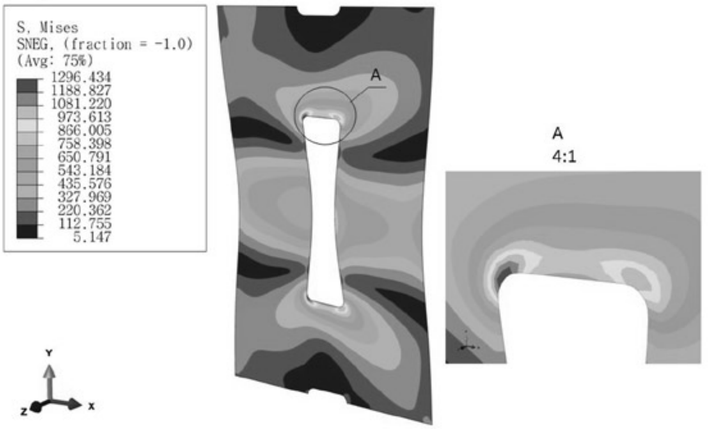

Fig. 7. Distribution of the H-M-H reduced stress in a plate with a 30x120mm cut-out, including the corners

An analysis of the plate's deformation reveals significant differences in the behaviour of particular regions of this structure. The vertical strips of the plate undergo bending, while the horizontal ones are subjected to torsion. Given such effort state of the plate, it is to be noted that structures made of isotropic materials will operate within a limited and, thus, unfavorable range due to exceeding the flow stress level in the cut-out corners. It seems highly probable that this undesired effect can be considerably reduced by the application of the material with orthotropic properties as it enables tailoring the material's operating characteristics as desired, depending on a region. Such properties are possessed, among others, by fibrous composites whose strength properties and rigidity can be shaped as required**,** hence they can be used to produce structures with improved load carrying capacity [4–7,26]. This problem will be investigated in further research.

6. Conclusions

The paper presented the numerical analysis of non-linear stability of composite plates subjected to compression. The results demonstrate that elastic properties of the investigated structures can be shaped to a great extent in the post-buckling state. The proposed solution regarding non-linear and elastic plates operating in a forced, higher buckling mode offers a broad spectrum of applications due to a relatively simple selection of the required parameters of the structure**,** such as its rigidity or the range of service loads. This is confirmed by the quantitative analysis of the results, particularly with regard to the maximum loads which range from 1003.1 to 5000 N depending on the cut-out size. We obtained an almost fivefold increase in load capacity in the elastic range for the plates with overall dimensions maintained identical. This observation is vital from the operating point of view, as it means that the structure's properties can be adjusted, while its designed structural features (dimensions) are retained and the weight is only slightly changed.

The findings are useful for examining structure deformation and effort in the full load range. As a result, it is possible to identify regions that are particularly prone to plastic deformation, which leads to loss of elastic properties of the structure. The numerical results reveal that these regions are located in the corners of the plate cut-out and that they determine the plate's load capacity in the elastic range. The analysis of the post-buckling equilibrium paths makes it possible to estimate the structure's behaviour after stability loss, which – in turn – enables the estimation of rigidity of the elastic element depending on cut-out sizes. Given the above, the results provide vital information for the forming process and optimization of the investigated structure's operating characteristics in terms of service loads.

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